Electric Charges and Fields

1. Electric Charge Charge is the property associated with matter due to which it produces and experiences electric and magnetic effect.
2. Conductors and Insulators Those substances which readily allow the passage of electricity through them are called conductors, e.g. metals, the earth and those substances which offer high resistance to the passage of electricity are called insulators, e.g. plastic rod and nylon.
3. Transference of electrons is the cause of frictional electricity.
4. Additivity of Charges Charges are scalars and they add up like real numbers. It means if a system consists of n charges q_1, q_2, q_3 , … ,q_n, then total charge of the system will be $q_1 + q_2 + \dots + q_n$.
5. Conservation of Charge The total charge of an isolated system is always conserved, i.e. initial and final charge of the system will be same.
6. Quantisation of Charge Charge exists in discrete amount rather than continuous value and hence, quantised.
Mathematically, charge on an object, $q=\pm ne$
where, n is an integer and e is electronic charge. When any physical quantity exists in discrete packets rather than in continuous amount, the quantity is said to be quantised. Hence, charge is quantised.
7. Units of Charge
(i) SI unit coulomb (C)
(ii) CGS system
(a) electrostatic unit, esu of charge or stat-coulomb (stat-C)
(b) electromagnetic unit, emu of charge or ab-C (ab-coulomb)
1 ab-C = 10 C, 1 C = 3×10^9 stat-C
8. Coulomb's Law It states that the electrostatic force of interaction or repulsion acting between two stationary point charges is given by

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

where, q_1 and q_2 are the stationary point charges and r is the separation between them in air or vacuum.

Also, $$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N-m}^2/\text{C}^2$$

where, ε_0 = permittivity of free space = 8.85419×10^{-12} $\text{C}^2/\text{N-m}^2$

The force between two charges q_1 and q_2 located at a distance r in a medium other than free space may be expressed as

$$F = \frac{1}{4\pi\varepsilon} \cdot \frac{q_1 q_2}{r^2}$$

where, ε is **absolute permittivity** of the medium.

Now, $$\frac{F_{\text{vacuum}}}{F} = \frac{\frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}}{\frac{1}{4\pi\varepsilon} \cdot \frac{q_1 q_2}{r^2}} = \frac{\varepsilon}{\varepsilon_0} = \varepsilon_r$$

where, ε_r is called **relative permittivity** of the medium also called **dielectric constant** of the medium.

In vector form,

$$\mathbf{F} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{|\mathbf{r}|^2} \hat{\mathbf{r}} \quad \text{or} \quad |\mathbf{F}| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

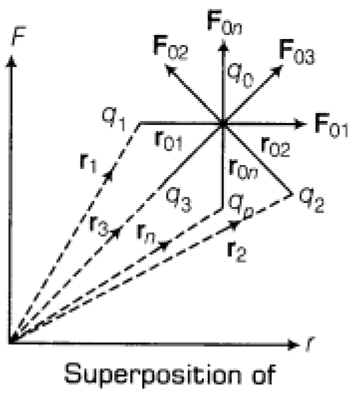
9. Electrostatic forces (Coulombian forces) are conservative forces.

10. Principle of Superposition of Electrostatic Forces This principle states that the net electric force experienced by a given charge particle q_0 due to a system of charged particles is equal to the vector sum of the forces exerted on it due to all the other charged particles of the system.

i.e. $\mathbf{F}_0 = \mathbf{F}_{01} + \mathbf{F}_{02} + \mathbf{F}_{03} + \ldots + \mathbf{F}_{0n}$

$$\mathbf{F}_0 = \frac{1}{4\pi\varepsilon_0} \left[\frac{q_1 q_0}{|\mathbf{r}_{01}|^3} \mathbf{r}_{01} + \frac{q_2 q_0}{|\mathbf{r}_{02}|^3} \mathbf{r}_{02} + \ldots + \frac{q_n q_0}{|\mathbf{r}_{0n}|^3} \mathbf{r}_{0n} \right]$$

where, $\mathbf{r}_{01} = \mathbf{r}_0 - \mathbf{r}_1$, $\mathbf{F}_{01}$ = force on q_0 due to q_1.



Superposition of electrostatic forces

Similarly, $\mathbf{r}_{0n} = \mathbf{r}_0 - \mathbf{r}_n$; $\mathbf{F}_{0n}$ = force on q_0 due to q_n

$$\therefore \qquad \mathbf{F}_0 = \frac{q_0}{4\pi\varepsilon_0}\left[\sum_{i=1}^{n} \frac{q_i}{|\mathbf{r}_{0i}|^3}\mathbf{r}_{0i}\right]$$

Net force in terms of position vector,

$$\mathbf{F}_0 = \frac{q_0}{4\pi\varepsilon_0}\left[\sum_{i=1}^{n} \frac{q_i}{|\mathbf{r}_0 - \mathbf{r}_i|^3}(\mathbf{r}_0 - \mathbf{r}_i)\right]$$

11. Electrostatic Force due to Continuous Charge Distribution

The region in which charges are closely spaced is said to have continuous distribution of charge. It is of three types given as below:

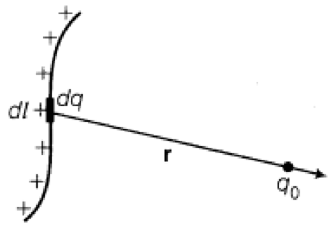
(*i*) Linear Charge Distribution

$$dq = \lambda\, dl$$

where, λ = linear charge density

$$d\mathbf{F} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_0\,(dq)}{|\mathbf{r}|^2}\,\hat{\mathbf{r}} \quad\Rightarrow\quad d\mathbf{F} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_0\,(\lambda\, dl)}{|\mathbf{r}|^2}\,\hat{\mathbf{r}}$$

Net force on charge q_0, $\qquad \mathbf{F} = \frac{q_0}{4\pi\varepsilon_0}\int_l \frac{\lambda\, dl}{|\mathbf{r}|^2}\,\hat{\mathbf{r}}$

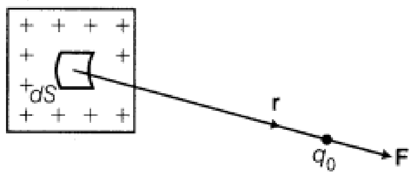


(*ii*) Surface Charge Distribution

$$dq = \sigma\, dS$$

where, σ = surface charge density

Net force on charge q_0, $\mathbf{F} = \frac{q_0}{4\pi\varepsilon_0}\int_S \frac{\sigma\, dS}{|\mathbf{r}|^2}\,\hat{\mathbf{r}}$

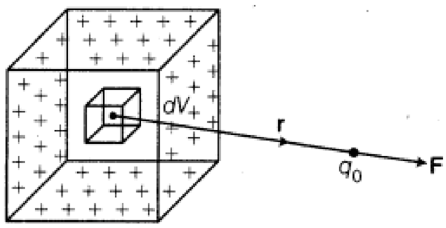


(*iii*) Volume Charge Distribution

$$dq = \rho\, dV$$

where, ρ = volume charge density

Net force on charge q_0, $\mathbf{F} = \frac{q_0}{4\pi\varepsilon_0}\int_V \frac{\rho\, dV}{|\mathbf{r}|^2}\,\hat{\mathbf{r}}$



12. Electric Field Intensity The electric field intensity at any point due to source charge is defined as the force experienced per unit positive test charge placed at that point without disturbing the source charge. It is expressed as

$$\mathbf{E} = \lim_{q_0 \to 0} \frac{\mathbf{F}}{q_0}$$

Here, $q_0 \to 0$, i.e. the test charge q_0 must be small, so that it does not produce its own electric field.

SI unit of electric field intensity $(\mathbf{E})$ is N/C and it is a vector quantity.

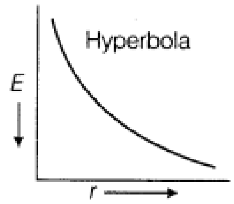
13. Electric Field Intensity (EFI) due to a Point Charge

Electric field intensity at P is, then

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{|\mathbf{r}|^2} \hat{\mathbf{r}}$$

The magnitude of the electric field at a point P is given by

$$|\mathbf{E}| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$



If $q > 0$, i.e. positive charge, then $\mathbf{E}$ is directed away from source charge. On the other hand if $q < 0$, i.e. negative charge, then $\mathbf{E}$ is directed towards the source charge.

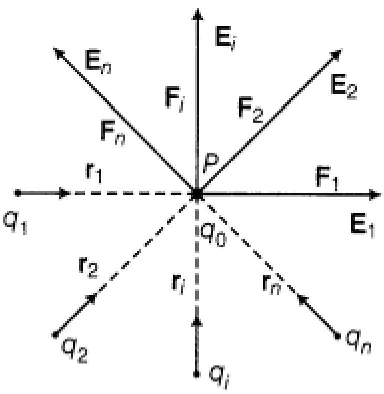
$$\mathbf{E} \propto \frac{1}{r^2}$$

14. Electric Field due to a System of Charges

Same as the case of electrostatic force, here we will apply principle of superposition, i.e.

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \dots + \mathbf{E}_n$$

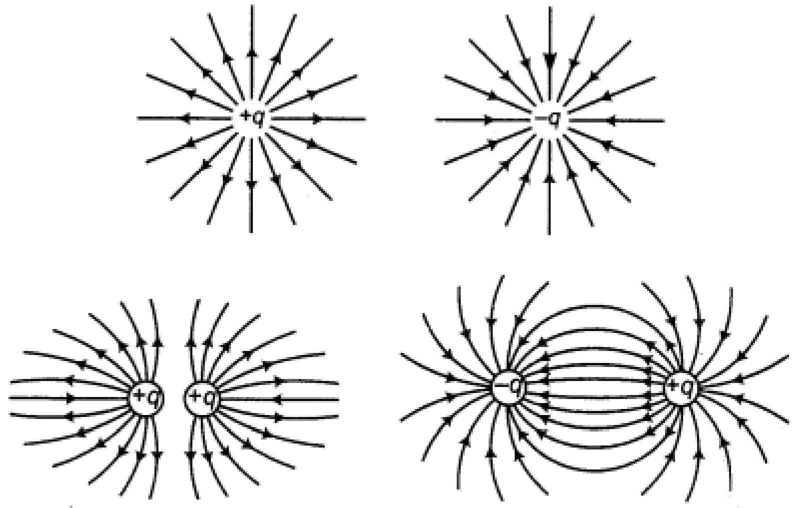
$$\Rightarrow \quad \mathbf{E} = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{|\mathbf{r}_i|^2} \hat{\mathbf{r}}_i$$



A system of charges

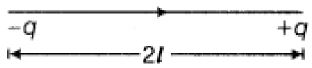
15. Electric Field Lines Electric field lines are a way of pictorially mapping the electric field around a configuration of charge(s). These lines start on positive charge and end on negative charge. The tangent on these lines at any point gives the direction of field at that point.

16. Electric field lines due **to positive and negative charge** and their combinations are shown as below:



Different electric field lines

17. Electric Dipole Two point charges of same magnitude and opposite nature separated by a small distance altogether form an electric dipole.
18. Electric Dipole Moment The strength of an electric dipole is measured by a vector quantity known as electric dipole moment (p) which is the product of the charge (q) and separation between the charges (2l).



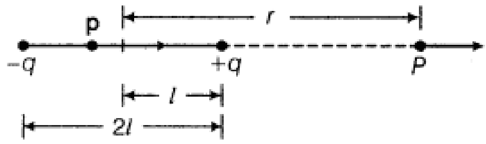
$$\mathbf{p} = q \times 2\mathbf{l}$$

$$\Rightarrow \quad |\mathbf{p}| = q\,(2l)$$

Direction Its direction is from negative charge $(-q)$ to positive charge $(+q)$.
SI unit Its SI unit is C-m.

NOTE The line joining the two charges $-q$ and $+q$ is called the dipole axis.

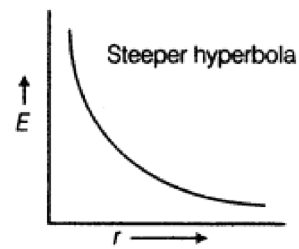
(i) Electric Field at any Point on the Axial Line/End-on Position of Electric Dipole



$$\mathbf{E}_{\text{axial}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2\mathbf{p}r}{(r^2 - l^2)^2}$$

When $l << r$, $\mathbf{E}_{\text{axial}} = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{2\,\mathbf{p}}{r^3} \Rightarrow |\mathbf{E}_{\text{axial}}| = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{2|\mathbf{p}|}{r^3}$

$$\mathbf{E} \propto \frac{1}{r^3}$$



The direction of electric field at any point on axial line is along the direction of electric dipole moment.

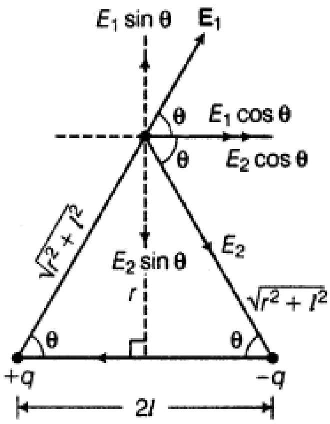
(*ii*) **Electric Field at any Point on Equatorial Line/Broadside on Position/Perpendicular Bisector of Electric Dipole**

$$\mathbf{E}_{\text{equatorial}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{-\mathbf{p}}{(r^2 + l^2)^{3/2}}$$

The direction of electric field intensity (**E**) due to dipole at any point on equatorial line is parallel to dipole and opposite to the direction of dipole moment.

If $l << r$,

$$|\mathbf{E}_{\text{equatorial}}| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{|\mathbf{p}|}{r^3}$$

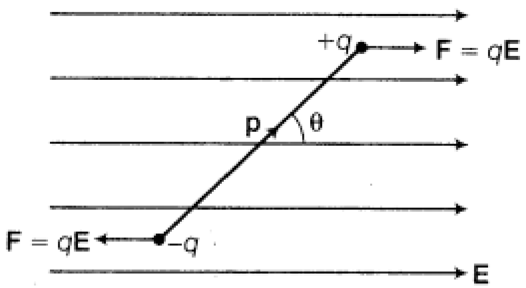


19. Electric Field due to a Dipole Electric field of an electric dipole is the space around the dipole in which the electric effect of the dipole can be experienced.

20. When $l << r$, $\dfrac{|\mathbf{E}_{\text{axial}}|}{|\mathbf{E}_{\text{equatorial}}|} = 2$

21. Torque on an electric dipole placed in a uniform electric field (E) is given by

$\tau = \mathbf{p} \times \mathbf{E} \Rightarrow |\tau| = pE\sin\theta$



22. **Minimum torque** experienced by electric dipole in electric field, when $\theta = 0°$ or π

$$\tau = \tau_{min} = 0$$

23. **Maximum torque** $\tau = \tau_{max}$, when $\sin\theta = 1 \Rightarrow \theta = \pi/2$

$$\tau_{max} = pE$$

24. Dipole is in stable equilibrium in uniform electric field when angle between p and E is 0° and in unstable equilibrium when angle θ= 180°.

25. Net force on electric dipole placed in a uniform electric field is zero.

26. There exists a net force and torque on electric dipole when placed in non-uniform electric field.

27. Work done in rotating the electric dipole from θ_1 to θ_2 is W = pE ($\cos\theta_1 - \cos\theta_2$)

28. Potential energy of electric dipole when it rotates from $\theta_1 = 90°$ to $\theta_2 = 0$

U = pE (cos 90° – cosθ) = -pE cos θ = – p .E

29. Work done in rotating the dipole from the position of stable equilibrium to unstable equilibrium, i.e. when $\theta_1 = 0°$ and $\theta_2 = \pi$.

W = 2 pE

30. Work done in rotating the dipole from the position of stable equilibrium to the position in which dipole experiences maximum torque, i.e. when $\theta_1 = 0°$ and $\theta_2 = 90°$.

W = pE