GRAVITATION

Preface

In the preparation of this chapter lot of efforts have been made to clear the concepts of Gravitation. When the topic has been completed relevant questions have been asked along with answer key to develop a transparent view of the topic. Having gone through the topic carefully only, you will be able to solve the problems of exercises. This topic is important and easy as concerned to the competitive examinations.

This book consists of theoritical & practical explanations of all the concepts involved in the chapter. Each article followed by a ladder of illustration. At the end of the theory part, there are miscellaneous solved examples which involve the application of multiple concepts of this chapter.

Students are advised to go through all these solved examples in order to develope better understanding of the chapter and to have better grasping level in the class.

Total number of Questions in **Gravitation** are :

In Chapter Examples 18

Solved Examples 25

Total no. of questions 43

1. KEPLER'S LAWS

Kepler found important regularities in the motion of the planets. These regularities are known as Kepler's three laws of planetary motion.

First Law : Every planet move around the sun in an elliptical orbit with sun at one of the focus. This is the law of orbits.

Second Law : The line joining the sun to the planet sweeps out equal area in equal interval of time i,e. the area velocity is constant. This is the law of area. The second law tells us that the planet will move most slowly when it is farthest from sun and most rapidly when it is nearest to sun. So we can say that this law is similar to law of conservation of angular momentum.

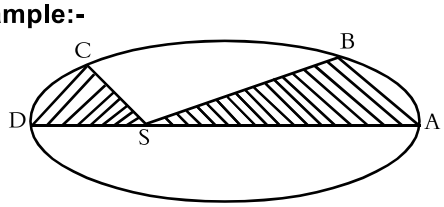
Where L = angular momentum

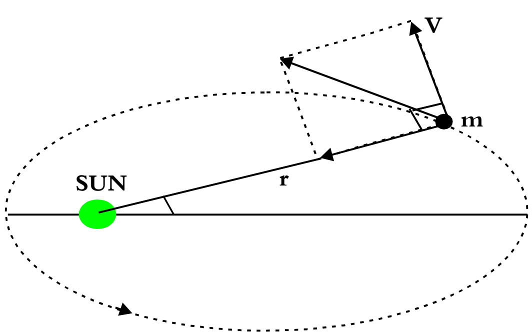
m = mass of planet

v = Linear velocity component $\perp$ to r

r = Distance of sun & planet.

Example:-





Areal velocity = $\frac{L}{2m} = \frac{vr}{2}$

Conclusions from the figure :

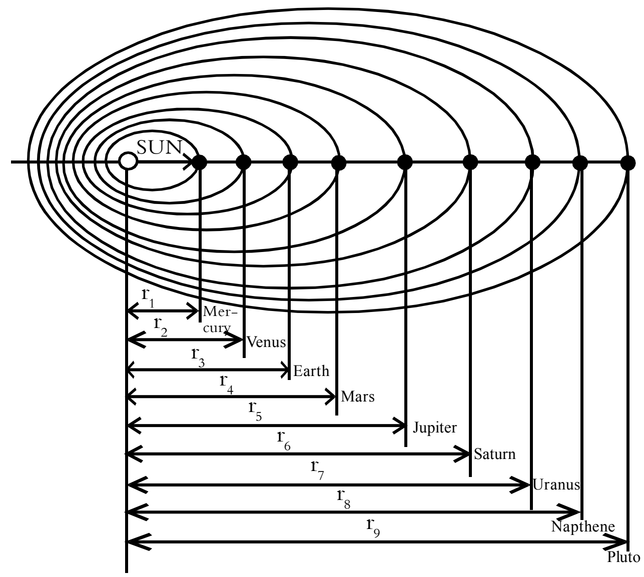
If a planet moves from A to B in time t_1, and from C to D in time t_2, then if

$t_1 = t_2$,Area of SBA = Area of SCD.

$t_1 > t_2$,Area of SBA > Area of SCD.

$t_1 < t_2$,Area of SBA < Area of SCD.

NOTE : When the planet is nearest to the sun, its speed is maximum and when it is farthest form the sun, then its speed is minimum.



Third Law : The square of the period of revolution (T) of any planet around the sun is directly proportional to the cube of its average distance (r) from the sun. This is the law of periods.

$T^2 \propto r^3$

or $T^2 = kr^3$,

where k = constant

Thus, it is clear from the law that larger the distance of a planet from the sun, larger will be its period of revolution around the sun.

NOTE :

a) Mercury is the nearest planet to the sun and its time period is 88 days while Pluto is the farthest planet and its time period is 248 years.

b) Kepler's laws are valid for motion of satellites.

c) For all the planets of the sun.

$$\frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3} = \frac{T_3^2}{r_3^3} = \text{--...--} = \text{constant}$$

d) When a planet comes close to the sun its linear velocity increases.

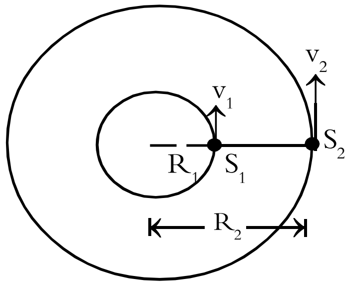
Examples based on Based on Keplar's Law

Ex.1 Two satellites S_1 and S_2 revolve round a planet in the same direction in circular orbits. Their periods of revolutions are 1 hour and 8 hour respectively. The radius of S_1 is 10^4 km. The velocity of S_2 with respect to S_1 will be-

(A) $\pi \times 10^4$ km/hr (B) $\pi/3 \times 10^4$ km/hr

(C) $2\pi \times 10^4$ km/hr (D) $\pi/2 \times 10^4$ km/hr

Sol. (A)



From Kepler's Law,

$$T^2 \propto r^3 \therefore \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$$

$$\Rightarrow \left(\frac{1}{8}\right)^2 = \left(\frac{10^4}{r_2}\right)$$

$$\Rightarrow \quad r_2 = 4\text{x } 10^4 \text{ km}$$

$$v = \omega r = \frac{2\pi r}{T}$$

$$\therefore |v_2 - v_1| = 2\pi\left(\frac{r_1}{T_1} - \frac{r_2}{T_2}\right)$$

$$= \pi \times 10^4 \text{ km/hr}$$

Ex.2 In the above example the angular speed of S_2 as actually observed by an astronaut in S_1 is -

(A) $\pi/3$ rad/hr (B) $\pi/3$ rad/sec
(C) $\pi/6$ rad/hr (D) $2\pi/7$ rad/hr

Sol **(A)**

When S_2 is closest to S_1, the speed of S_2 relative to S_1 is $v_2 - v_1 = \pi \times 10^4$ km/hr. The angular speed of S_2 as observed from S_1 (when closest distance between them is $r_2 - r_1 = 3 \times 10^4$ km)

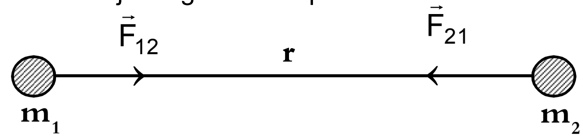
$$\omega = \frac{v_2 - v_1}{r_2 - r_1} = -\frac{\pi \times 10^4}{3 \times 10^4}$$

$$= -\frac{\pi}{3}\text{rad/hr}, \ |\omega| = \frac{\pi}{3}\text{rad/hr}$$

2. NEWTON'S LAW OF GRAVITATION

a) Every two objects in the universe attract each other. This force of attraction is called *'Gravitational force'*.

b) The force of attraction between any two material particles is directly proportional to the product of the masses of the particles and inversely proportional to the square of the distance between them. It acts along the line joining the two particles.



$$\therefore F \propto m_1 m_2$$

and $$F \propto \frac{1}{r^2}$$

or $$F = \frac{G m_1 m_2}{r^2}$$

c) G is the constant of proportionality which is called 'Newton's gravitation constant.

$G = 6.67 \times 10^{-11}$ Nm^2 / kg^2

$G = 6.67 \times 10^{-8}$ dyne cm^2 / gm^2

d) Dimensional formula of G is $[M^{-1} L^3 T^{-2}]$

e) In vector form-

The force exerted by point mass (2) on point mass (1) will be -

$$\vec{F}_{12} = \frac{G m_1 m_2}{r_{12}^2} \hat{r}_{12}$$

direction of $\hat{r}_{12}$ is from 1 to 2.

$$\vec{F}_{21} = \frac{G m_1 m_2}{r_{12}^3} s \ \vec{r}_{21} .,$$

similarly $$\vec{F}_{12} = -\frac{G m_1 m_2}{r_{21}} \vec{r}_{21}.$$

f) $$\vec{F}_{21} = -\vec{F}_{12}$$

but, $$|\vec{F}_{21}| = |\vec{F}_{12}|$$

From above two expression we can conclude that the force exerted between two particles is equal in magnitude but opposite in direction.

g) Gravitational force is the weakest force in nature.

NOTE :

The ratio of gravitational to electrostatic force between two electrons is of the order.

$$\frac{F_g}{F_e} = 10^{-43}.$$

h) The range of this force is maximum upto infinity.

i) It is due to very small value of G that we do not experience the gravitational force in our daily life. But masses of celestial bodies are so large that the magnitude of the force of attraction between them is appreciable. In the motion of planets and satellites, this force supplies the necessary centripetal force due to which earth revolves around the sun and moon around the earth.

j) If the density of the earth is assumed to be uniform and a particle moves inside the earth then the gravitational force decreases because the shell of the material lying outside the particle's radial position would not exert any force on the particle.

Examples based on Newton's Law of Gravitation

Ex.3 A mass M splits into two parts m and (M – m), which are then separated by a certain distance. What ratio (m / M) maximises the gravitational force between the parts. ?

(A) $\frac{2}{3}$ (B) $\frac{3}{4}$

(C) $\frac{1}{2}$ (D) $\frac{1}{3}$

Sol. (C)

If r is the distance between m and (M – m), the gravitational force will be -

$$F = G\frac{m(M-m)}{r^2} = \frac{G}{r^2}(mM - m^2)$$

The force will be maximum if, $\frac{dF}{dm} = 0$

i.e $\frac{d}{dm}\left[\frac{G}{r^2}(mM - m^2)\right] = 0$

or $\frac{m}{M} = \frac{1}{2}$ (as M and r are constants)

3. GRAVITATIONAL AND INERTIAL MASS

(a) Inertial mass : When mass is defined on the property of inertia, it is termed as inertial mass -

(b) Gravitational mass : When mass is defined on the property of gravity, it is called gravitational mass.

(c) Properties of Inertial mass :

i) It is equal to the ratio of magnitude of external force applied on the body to the accleration produced in it by that force.

$m = F/a$.

ii) It is proportional to the quantity of matter present in the body.

iii) It is independent of shape, size and state of the body.

iv) It is not affected by the presence of other bodies near it.

v) When various masses are put together, the inertial masses add according to the scalar laws irrespective of the material of the bodies involved.

vi) It increases with increase in velocity according to the relation $m = \frac{m_0}{\sqrt{1-(v^2/c^2)}}$, where m_o = rest mass of the body. C = velocity of light and v is the velocity of particle.

Point to remember :

i) It is found that the ratio of two gravitational masses is same as the ratio of their inertial masses.

ii) If a ball is dropped from the hole passing though the two poles and the centre of earth then it will do S.H.M. in the tunnel.

Examples Based on Gravitational inertial mass

Ex.4 If an object having mass $\sqrt{3}$ kg in moving with a velocity that is $\frac{1}{2}$ the velocity of light then inertial mass will be -

(A) 2 kg (B) 1 kg

(C) $\sqrt{3}$ kg (D) 0 kg

Sol $m_g = \sqrt{3}$ kg , $v = \frac{c}{2}$

$$m = \frac{m_0}{\sqrt{1-(v^2/c^2)}} = \frac{\sqrt{3}}{\sqrt{1-\frac{c^2}{4xc^2}}}$$

$$= \frac{\sqrt{3}}{\frac{\sqrt{3}}{2}} \text{ kg}$$

4. ACCELERATION DUE TO GRAVITY

a) Acceleration produced in a body due to the force of gravity is termed as acceleration due to gravity.

b) The acceleration due to gravity is the rate of increase of velocity of a body falling towards the earth.

c) The acceleration due to gravity is equal to the force by which earth attracts a body of unit mass towards its centre.

d) Let 'm' be the mass of body and 'F' be the force of attraction at a distance 'r' from the centre of earth, then acceleration due to gravity (g) at that place will be.

$$g = \frac{F}{m} = \frac{GM_e}{r^2}$$, where M_e = mass of earth.

e) The expression $g = \frac{GM_e}{r^2}$ is free from 'm' (mass of body). This means that the value of 'g' does not depend upon the shape, size and mass of the body. Hence if two bodies of different masses, shapes and sizes are allowed to fall freely, they will have the same acceleration. If they are allowed to fall form the same height, they will reach the earth simultaneously.

f) The acceleration of a body on the surface of the earth is g= 9.80 m/s^2 or 981 cm/s^2.

g) Dimensional formula of g is $[M^0 L^1 T^{-2}]$.

h) The value of acceleration due to gravity depends on the following factors.

a) Height above the earth surface.

b) Depth below the earth surface.

c) Shape of the earth.

d) Axial rotation of the earth.

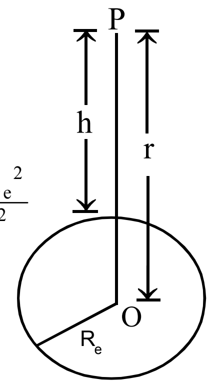
(a) Height above the surface of earth :

(i) As we go above the surface of the earth, the value of 'g' decreases.

Consider a point P at a distance r form the centre of earth.

The acceleration due to gravity at point P is.

$$g' = \frac{g}{\left(1+\frac{h}{R_e}\right)^2}$$



$$\Rightarrow g' = g\left(\frac{R_e}{R_e+h}\right)^2 = \frac{gR_e^2}{r^2}$$

$\therefore$ g' < g

(ii) As we go above the surface of the earth, the value of 'g' decreases. $g' \propto \frac{1}{r^2}$ for $r > R_e$

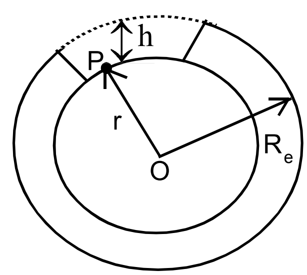
(iii) If $h << R_e$, (according to binomial expansion) $g' = g\left(1-\frac{2h}{R_e}\right)$

(iv) If $r = \infty$, $g' = 0$. At infinite distance form the earth, the value of 'g' becomes zero.

(v) Value of g at the surface of earth (h = 0)

$$\Rightarrow g = \frac{GM_e}{R_e^2}$$

(b) Below the surface of earth :



(i) The value of 'g' decreases on going below the surface of the earth.

(ii) The value of 'g' at a distance h below this earth's surface be g_h and 'g' at the earth's surface then -

$$g_h = g\left(1-\frac{h}{R_e}\right) = g\ (R_e-h)/R_e = \frac{gr}{R_e}$$

i.e $g_h < g$

r is the distance form the centre of the earth ($r < R_e$), $r = R_e - h$.

(iii) If d is the density of the earth then the force on pt. P is

$g_h = \frac{M_r G}{r^2}$ where $M_r = \frac{4}{3}\pi (R_e - h)^3 d$;

$r^2 = (R_e - h)^2 \Rightarrow g_h = \frac{4}{3}\pi G (R_e - h)d$

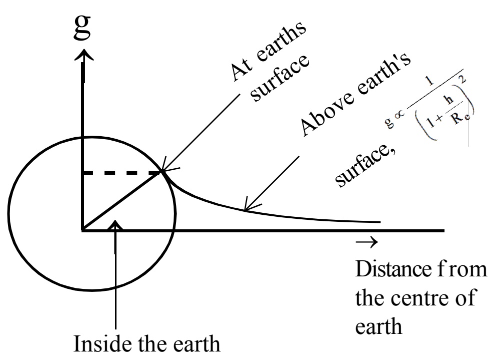
(iv)) At the centre of the earth,

$h = R_e$ (i.e $r = 0$) so $g = 0$.

(v) Value of 'g' is maximum at the surface of earth.

(vi) Graphical representation of variation in the value of g

Variation of g with distance r from the centre of earth

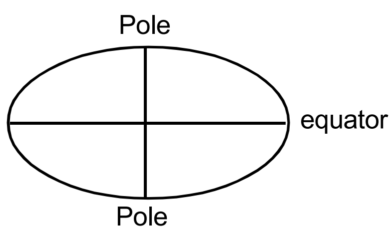


(c) Variation in value of 'g' on the surface of earth :

It is due to two reasons :

(1) Due to shape of earth :

(i) The earth is elliptical in shape. It is flattened at the poles and bulged out at the equator. Now, we know that $g \propto 1/R_e^2$, therefore the value of g at the equator is minimum and the value of g at the poles is maximum ($\because$ Radius at poles is < Radius at equator,)



$$g_e = \frac{GM}{R_e^2} \quad \text{(At equator)}$$

$$g_p = \frac{GM}{R_p^2} \quad \text{(At pole)}$$

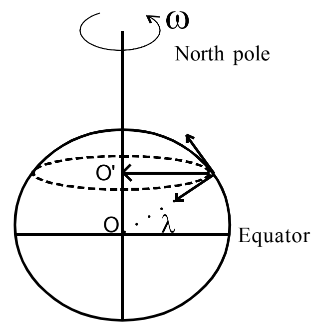
$\because \quad R_e > R_p$

Hence, $g_e < g_p$

(ii) $\frac{g_p}{g_e} = \frac{R_e^2}{R_p^2}$

(iii) Numerical value of R_p is twenty one kilometre less than R_e. Therefore $g_p - g_e = 0.02 m/s^2$.

(2) Due to rotation of earth :



(i) Earth is rotating about its own axis form west to earth with an angular velocity ω.

(ii) On a latitude λ, point P is moving in circle with radius 'r'. If we keep a body then some part of its gravita tional force will be used up for providing centripetal force, therefore there is reduction in total gravitational force. As a result of this, value of 'g' decreases.

(iii) If ω is the angular velocity of rotation of the earth, R_e is radius of the earth, then the observed value of g at the latitude λ is represented by g' then.

$$g' = g_o - \omega^2 R_e \cos^2 \lambda$$

or $g' = g_o - 0.0337 \cos^2 \lambda$

where g_o is value of 'g' at the poles.

(iv) At equator, $\lambda = o$

$g' = g_0 - \omega^2 R_e$ (Minimum value)

$= g - 0.0337$

(v) At poles, $\lambda = 90^o$

$\therefore \quad \cos \lambda = 0$

$g' = g$ (Maximum value)

(vi) From the above expressions we can conclude that the value of 'g' at the surface of earth is maximum at poles and minimum at the equator. Therefore weight of bodies is maximum at the poles and will go on decreasing towards the equator. (it is minimum at the equator).

(vii) If earth stops rotating about its axis ($\omega = o$), the value of g will increase everywhere, except at the poles. On the contrary, if there is increase in the angular velocity of earth, then except at the poles the value of 'g' will decrease at all places.

(viii) Maximum effect of rotation takes place at the equator while at poles, there is no effect.

(ix) If $\omega = \sqrt{\frac{g}{R_e}}$ then, at equator weight of body will become zero but its mass remains unaltered.

a) That means if the earth starts rotating with an angular speed 17 times the present,

b) If $g_{equator} = 0$, in this condition , time period of earth's rotation will become 1.41 hours instead of 24 hours.

Examples based on Accelaration due to gravity

Ex.5 What would be the angular speed of earth, so that bodies lying on equator may appear weightless ?

($g = 10 m/s^2$ and radius of earth = 6400 km)

(A) 1.25×10^{-3} rad/sec
(B) 1.25×10^{-2} rad/sec
(C) 1.25×10^{-4} rad/sec
(D) 1.25×10^{-1} rad/sec

Sol. (A)

$g' = g - R_e\omega^2$ (at equator $\lambda = 0$)

If a body is weightless,

$g' = 0$, $g - R_e\omega^2 = 0$

$$\Rightarrow \omega = \sqrt{\frac{g}{R}} = \sqrt{\frac{10}{6400 \times 10^3}}$$

$= 1.25 \times 10^{-3}$ rad/sec.

Ex.6 The speed with which the earth have to rotate on its axis so that a person on the equator would weight (3/5) th as much as present will be - (Take the equatorial radius as 6400 km.)

(A) 3.28×10^{-4} rad/sec
(B) 7.826×10^{-4} rad/sec
(C) 3.28×10^{-3} rad/sec
(D) 7.28×10^{-3} rad/sec

Sol. (B)

The apparent weight of person on the equator (latitude $\lambda = 0$) is given by

$$W' = W - m R_e\omega^2,$$

$$W' = \frac{3}{5} W = \frac{3}{5} mg$$

$$\frac{3}{5} mg = mg - mR\omega^2 \text{ or}$$

$$mR\omega^2 = mg - \frac{3}{5} mg$$

$$\omega = \sqrt{\frac{2}{5}\frac{g}{R}} = \sqrt{\frac{2}{5} \times \frac{9.8}{6400 \times 10^3}} \text{ rad/ sec}$$

$= 7.826 \times 10^{-4}$ rad/sec

Ex.7 On a planet (whose size is the same as that of earth and mass 4 times to the earth) the energy needed to lift a 2kg mass vertically upwards through 2m distance on the planet is ($g = 10 m/sec^2$ on surface of earth) -

(A) 16 J (B) 32 J
(C) 160 J (D) 320 J

Sol. **(C)**

According to question,

$g' = \frac{G \times 4M_p}{R_p^2}$ on the planet and $g = \frac{GM_e}{R_e^2}$ on the earth

$\because$ $R_p = R_e$ and $M_p = M_e$

Now $\frac{g'}{g} = 4 \Rightarrow g' = 4g = 40 \text{ m/sec}^2$

Energy needed to lift 2 kg mass through 2m distance = $mg'h = 2 \times 40 \times 2 = 160$ J

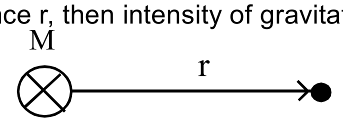
5. GRAVITATIONAL FIELD AND GRAVITATIONAL FIELD INTENSITY

a) It is defined as the space around the attracting body, in which its attraction (gravitational) can be experienced.

b) ***Intensity of gravitational field or gravitational field strength*** - It is defined as the force experienced by unit mass place at any point in the gravitational field.

c) Gravitational field is a vector quantity.

d) Suppose a body of mass M is placed at a distance r, then intensity of gravitational field



at point P will be $\vec{E} = \frac{GM_e}{r^2} (-\hat{r})$

e) Unit of gravitational field strength is Newton/ kg or m/sec^2.

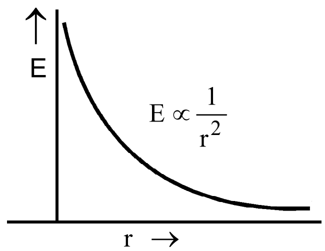
Dimension formula is $[M^0L^1T^{-2}]$.

f) As the distance (r) increases, gravitational field strength decreases.
At r = ∞, value of intensity of gravitational field becomes zero.

g) Intensity of gravitational field at a distance r form the centre of earth is

$$E = \frac{GM_e}{r^2} = g$$

NOTE : From this expression



$$E = \frac{GM_e}{r^2} = g$$

it is clear that the intensity of gravitational field at any place is
equal to acceleration due to gravity.

h) Change of intensity of gravitational field due to a point mass with respect to distance -

$$E = \frac{GM_e}{r^2}$$

i) Relation between gravitational field and gravitational potential -

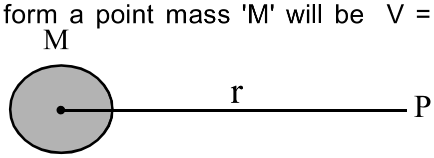
$$E = -\nabla V$$

$$E = -\frac{dV}{dr}$$

6. GRAVITATIONAL POTENTIAL

a) The work done in bringing a unit mass form infinity to a point in the gravitational field is called the 'gravitational potential' at that point.

b) Gravitational potential at a point P distance r form a point mass 'M' will be $V = -\frac{GM}{r}$



c) Unit of Gravitational potential is Joule/kg

d) Dimensional formula of Gravitational potential is $[M^o L^2 T^{-2}]$.

e) Gravitational potential is a scalar quantity.

f) At r = ∞, V = 0.

Examples based on Gravitational Potential

Ex.9 Two bodies of mass 10^2 kg and 10^3 kg are lying 1m apart. The gravitational potential at the mid-point of the line joining them is -

(A) 0

(B) –1.47 Joule/kg

(3) 1.47 Joule/kg

(4) -147×10^{-7} Joule /kg

Sol **(4)**

$$V_g = V_{g1} + V_{g2} = -\frac{Gm_1}{r_1} - \frac{Gm_2}{r_2}$$

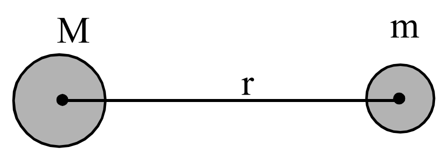
$$= -6.67 \times 10^{-11}$$

$$= -1.47 \times 10^{-7} \text{ Joule/kg}$$

7. GRAVITATIONAL POTENTIAL ENERGY

a) The gravitational potential energy of a body at a point is defined as the amount of work done in bringing the body from infinity to that point against the field.

b) The gravitational potential energy of mass 'm' in the gravitational field of mass M at a distance r form it is



$$U = -\frac{GMm}{r}$$

where r is distance between M and m.

c) At any place in gravitational field, gravitational potential is V, then the gravitational potential energy of a mass 'm' at that place will be -
U = – mV.

d) The gravitational potential energy of a particle of mass 'm' at a point distant 'r' from the centre of earth is

$$U = -\frac{GM_e m}{r}, \quad \text{if } r > R_e$$

$$= -\frac{GMem(3R_e^2 - r^2)}{2R_e^3}, \quad \text{if } r < R_e$$

e) Force between two particle if their potential energy is U, is $F = -\frac{dU}{dr} = -\frac{d}{dr}\left(-\frac{GMm}{r}\right)$

$= -\frac{GMm}{r^2}$ minus sign. indicates that the force on the bodies is towards each other.

Note :- If a particle is at a height h from earth's surface and R_e be the radius of earth, $r = R_e + h$

$$U = -\frac{GM_e m}{R_e + h}$$

f) It is a scalar quantity and its value is always negative.

g) It's unit is Joule or Erg.

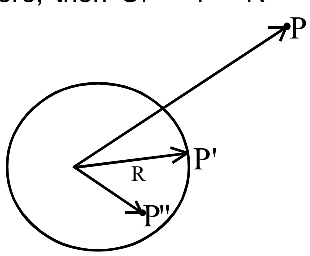
h) Gravitational potential energy of a mass at infinite distance from earth is zero, and at all the other points it is less than zero, i.e. it is negative.

7.1 Intensity of Gravitational field and gravitational potential due to hollow sphere

a) Hollow Sphere :

(i) Let OP = r

If point P is situated outside the hollow sphere, then OP = r > R



(1) $E_{out} = -\frac{GM}{r^2}$ (2) $V_{out} = -\frac{GM}{r}$

(ii) If point P is situated on the surface of sphere, then OP' = r = R

(1) $E_{surface} = -\frac{GM}{R^2}$ (2) $V_{surface} = -\frac{GM}{R}$

(iii) If point P is inside the hollow sphere, then OP" = r < R

(1) $E_{in} = 0$ (2) $V_{in} = -\frac{GM}{R}$

Note : Gravitational field intensity inside a hollwo sphere is zero but gravitational potential is constant and is equal to the potential at the surface.

b) Solid Sphere :

Let OP = r

(i) If point P is situated outside the sphere, then OP = r > R

(1) $E_{out} = -\frac{GM}{r^2}$ (2) $V_{out} = -\frac{GM}{r}$

(ii) If point P is situated on the surface of sphere, then OP = r = R

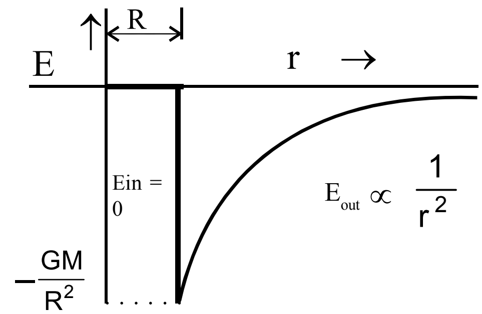
(1) $E_{surface} = -\frac{GM}{R^2}$ (2) $V_{surface} = -\frac{GM}{R}$

(iii) If point P is situated inside the sphere, then OP = r < R_e

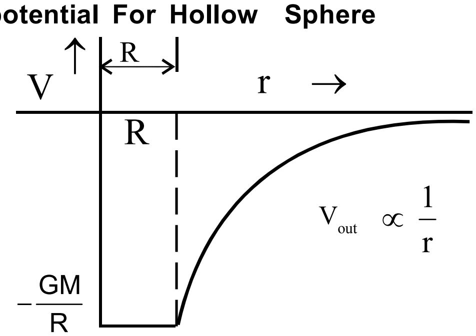
(1) $E_{in} = -\frac{GMr}{R^3}$ (2) $V_{in} = -\frac{GM(3R^2 - r^2)}{2R^3}$

Note : $V_{centre} = 1.5\ V_{surface}$

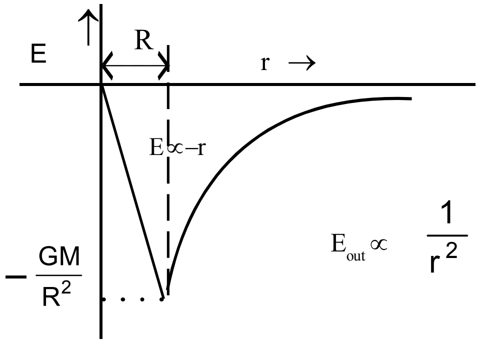
Graphical Represenation of Gravitational Field Intensity For Hollow Sphere



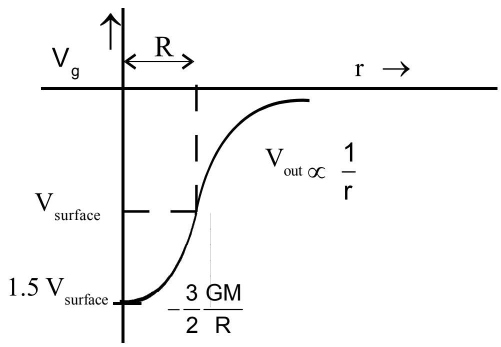
Graphical Represenation of Gravitational potential For Hollow Sphere



For Solid Sphere



For Solid Sphere



Examples based on Gravitational Potential Energy

Ex.10 If g is the acceleration due to gravity on the earth's surface, the gain in P.E. of an object of mass m raised from the surface of the earth to a height of the radius R of the earth is -

(A) mgR (B) 2mgR

(C) $\frac{1}{2}$ mgR (D) $\frac{1}{4}$ mgR

Sol. (C)

The P.E. of the object on the surface of earth is $U_1 = -\frac{GMm}{R}$

The P.E. of object at a height R,

$U_2 = -\frac{GMm}{(R+R)}$ The gain in P E is $U_2 - U_1$

$= \frac{GMm}{2R} = \frac{1}{2}$ mgR

$\left[\because g = \frac{GM}{R^2} \text{on surface of earth}\right]$

8. SATELLITE

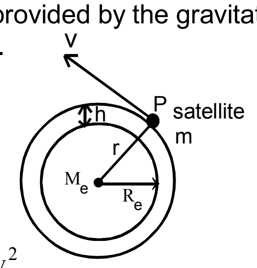
a) Celestial bodies revolving round the gravitational field of the planets is called satellite.

b) Satellites are of two types -

a) Natural satellites - As moon is a satellite of the earth.

b) Artificial satellites - They are launched by man such as Rohini, Aryabhatt etc.

c) Let a satellite of mass 'm' revolves in a circular orbit with radius 'r' around the earth. The necessary centripetal force needed for circular motion is provided by the gravitational force of the earth.



so, $\frac{GM_e m}{r^2} = \frac{mv^2}{r}$

Where M_e = mass of earth

v = Orbital velocity of satellite

r = Radius of satellite's orbit = R_e + h = orbital radius

R_e = Radius of earth

h = The height of the satellite above the earth's surface

g = Acceleration due to gravity on the surface of the earth.

d) **Orbital velocity of satellite :**

i) $v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM_e}{R_e + h}} = \sqrt{\frac{gR_e^2}{R_e + h}}$

ii) From this expression, $\sqrt{\frac{gR_e^2}{R_e + h}}$ ' it is clear that orbital velocity does not depend on the mass of satellite but it depends on the height of the satellite above the earth's surface (h). Greater the height of satellite, smaller is the orbital velocity.

Note :- This proves that keplers II Law is true.

iii) If a satellite is very close to the earths surface (h << Re), then h will be negligible as compared to R than the orbital speed of satellite is given by

$v = \sqrt{\frac{GM_e}{R}} = \sqrt{gR}$

= 7.92 km/sec. (≃ 8km/sec.).

e) Period of Revolution :

i) The time taken by the satellite for completing one revolution of earth is called as period of revolution of satellite.

ii) Period of Revolution of a satellite is

$$T = \frac{2\pi r}{v} = \frac{2\pi(R_e + h)}{v}$$

where 'T' is the time period of a satellite at a height 'h'.

iii) $T = 2\pi\sqrt{\frac{r^3}{GM_e}} = 2\pi\sqrt{\frac{r^3}{gR_e^2}}$

$$= 2\pi\sqrt{\frac{(h+R_e)^3}{gR_e^2}} = 2\pi\sqrt{\frac{R_e}{g}}\left(1+\frac{h}{R_e}\right)^{3/2}$$

iv) It is evident from the above expression that $T^2 \propto r^3$ i.e kepler's III law is true for circular motion also.

v) For a satellite revolving very close to the surface of earth ($h << R_e$),

$$T = 2\pi\sqrt{\frac{R_e}{g}} = 84.4 \text{ min.}$$

therefore, the minimum time period of the satellite revolving very close to the surface of earth is 84.4 min.

vi) From the expression '$T = 2\pi\sqrt{\frac{r^3}{GM_e}}$ we can say that time period of satellite depends on its orbital radius $T^2 \propto r^3$

As the radius increases, simultaneously time period also increase.

f) Energy of Satellite : When satellite is revolving in the orbit of radius 'r' then

i) Potential energy of satellite :

$$P.E = -\frac{GM_e m}{r}, \text{ where } r = h$$

ii) Kinetic energy of satellite :

$$K.E = \frac{1}{2}mv^2 = \frac{GM_e m}{2r}$$

iii) Total energy of satellite

$$= K.E + P.E = \frac{-GM_e m}{2r}$$

iv) We can say that : Total energy of satellite

$= \frac{1}{2}$(Potential energy of satellite)

= – Kinetic Energy of satellite.

g) Binding Energy of satellite :

(i) Binding energy is the energy given to satellite in order that the satellite escape away form its orbit.

Binding Energy =

$$- \text{Total Energy} = \frac{GM_e m}{2r}$$

(i.e equal to kinetic energy)

If energy equals to $\frac{GM_e m}{2r}$, is provided to the satellite, it will escape away from the gravitational field of the planet.

(ii) Unless a revolving satellite gets extra energy, it would not leave its orbit. If the kinetic energy of a satellite happens to increase to two times, the satellite would escape.

(iii) If the orbital velocity of a satellite revolving close to the earth happens to increase to $\sqrt{2}$ times, the satellite would escape. That means if orbital velocity increases to 41.4% , satellite would leave the orbit.

(iv) Total energy of satellite is always negative. When the energy of the satellite is negative, it moves in either a circular or an elliptical orbit.

(v) Binding Energy = Kinetic Energy =

$$-(\text{Total Energy}) = -\frac{(\text{Potential Energy})}{2}$$

Examples based on Satellite

Ex.11 The mass and radius of earth and moon are M_1, R_1 and M_2, R_2 respectively. Their centres are d distance apart. With what velocity should a particle of mass m be projected from the mid point of their centres so that it may escape out to infinity -

(A) $\sqrt{\frac{G(M_1+M_2)}{d}}$ (B) $\sqrt{\frac{2G(M_1+M_2)}{d}}$

(C) $\sqrt{\frac{4G(M_1+M_2)}{d}}$ (D) $\sqrt{\frac{GM_1M_2}{d}}$

Sol. (C)

Total energy of the particle at P

$E = E_{kP} + U$

$$= \frac{1}{2}mv_e^2 - \frac{GM_1m}{d/2} - \frac{GM_2m}{d/2}$$

$$= \frac{1}{2}mv_e^2 - \frac{2Gm}{d}(M_1 + M_2)$$

At infinite distance from M_1 and M_2, the total energy of the particle is zero.

$$\therefore \frac{1}{2} mv_e^2 = \frac{2Gm}{d}(M_1 + M_2),$$

$$\therefore v_e = \sqrt{\frac{4G}{d}(M_1 + M_2)}$$

Ex.12 A satellite has to revolve round the earth in a circular orbit of radius 8 x 10^3 km. The velocity of projection of the satellite in this orbit will be -

(A) 16 km/sec (B) 8 km/sec
(C) 3 km/sec (D) 7.08 km/sec

Sol. (D)

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{gR^2}{r}} = \sqrt{\frac{9.8 \times 6.4^2 \times 10^{12}}{8 \times 10^6}}$$

= 7.08 km/sec.

Ex.13 The moon revolves round the earth 13 times in one year. If the ratio of sun-earth distance to earth-moon distance is 392, then the ratio of masses of sun and earth will be -

(A) 365 (B) 356
(C) 3.56×10^5 (D) 1

Sol. (C) Period of revolution of earth around sun

$$T_e^2 = \frac{4\pi^2 R_e^2}{GM_s}$$

Period of revolutions of moon around earth

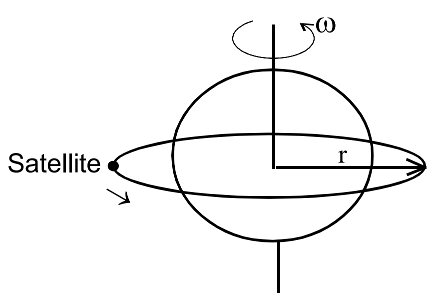
$$T_n^2 = \frac{4\pi^2 R_m^2}{GM_e}$$

$$\therefore \left(\frac{T_e}{T_m}\right)^2 = \left(\frac{M_e}{M_s}\right)\left(\frac{R_e}{R_m}\right)^3$$

$$\therefore \frac{M_s}{M_e} = \left(\frac{T_m}{T_e}\right)^2 \left(\frac{R_e}{R_m}\right)^3$$

$$= \frac{(392)^3}{13^2} = 3.56 \times 10^5$$

9. GEO-STATIONARY SATELLITES



a) Such satellites which are stationary with respect to an observer on earth are termed as Geostationary satellites. They are also called Parking satellites.

b) The direction of rotation of geo-stationary satellites is from west to east, the time period is 24 hours and its angular velocity is same as that of axial velocity of earth, revolving around its axis.

c) Geo-stationary satellites can be launched just above the equator.

d) The radius of orbit of Geo-stationary satellite is r = 42,000 km and

its height above the surface of earth is h = 36, 000 km.

e) Different values of satellite -

a) Angular Velocity (ω) = 7.1 x 10^{-5} rad/sec

b) Linear Velocity (v) = 3.1 km/sec.

c) Time Period (T) = 24 hours.

d) Height above the earth's surface (h) = 36,000km (approx.)

f) At time t, the angular displacement of earth and Geo stationary satellite is same.

g) Angular momentum of satellite is conserved and it is equal to

$$J = mvr = mr\sqrt{\frac{gR_e^2}{r}} = mR_e\sqrt{gr} = m\sqrt{GM_e r}$$

h) Satellites behaves like freely falling bodies towards planet.

i) The satellite revolves around the earth in an orbit with earth as centre or a focus.

j) If a packet is released form the satellite, it will not fall on the earth but will remain revolving in the same orbit with the same speed as the satellite.

k) No gravitational force of satellite is used up for providing necessary centripetal force. Due to gravitational force the effective value of acceleration due to gravity becomes g_{eff} = 0, as a result effective weight becomes w_{eff} = 0, so the man sitting in the satellite enjoys weightlessens. Man experiences this weightlessness condition only when weight of satellite is very less therefore gravitational effect of satellite is negligible.

Although moon is also a satellite of the earth, but a person on moon does not feel weight-lessness. Thereason is that the moon has a large mass and exerts a gravitational force on the person (and this is the weight of the person on the moon). On the other hand, the artificial satellite having a smaller mass does not exert gravitational force on the space-man.

9.1 Relation between velocity of Projection and shape of orbit

Shape of the satellite's orbit depends on its velocity.

$V_0 = \sqrt{\frac{GM_e}{R_e + h}}$; orbital velocity of the satellite's

Cases :

(a) **If $V < V_0$;** In this case satellite will leave its circular orbit and finally fall to earth following spiral path.

(b) **If $V = V_0$;** In this case satellite will rotate in circular path.

(c) **If $V_0 < V < \sqrt{2}\, V_0$;** In this case satellite will revolve around the earth in elliptical orbit.

(d) **If $V = \sqrt{2}\, V_0$;** In this case satellite will leave the gravitational field of earth and escape away following a parabolic path.

(e) **If $V > \sqrt{2}\, V_0$;** In this case the satellite will escape, following a hyperbolical path.

Examples based on Geo- Stationary Satellites

Ex.14 The earth is revolving round the sun in an elliptical orbit. If $\frac{OA}{OB}$ = x, the ratio of speeds of earth at B and A will be -

(A) x (B) $\sqrt{x}$

(C) x^2 (D) $x\sqrt{x}$

Sol. (A)

According to law of conservation of angular momentum,

mvr = constant

$\Rightarrow$ vr = constant

$v_{max} . r_{min} = v_{min} . r_{max}$

$$\Rightarrow \frac{V_B}{V_A} = \frac{v_{max}}{v_{min}} = \frac{r_{max}}{r_{min}} = x$$

Ex.15 A satellite of mass m is revolving in a circular orbit of radius r. the relation between the angular momentum J of satellite and mass m of earth will be -

(A) $J = \sqrt{G.Mm^2r}$ (B) $J = \sqrt{GMm}$

(C) $J = \sqrt{GMmr}$ (D) $J = \sqrt{\frac{mr}{M}}$

Sol. Angular momentum of satellite, J = mvr. But,

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{GM}{r}}$$

$\therefore$ $J = m\sqrt{GMr}$

10. ESCAPE VELOCITY

a) Escape velocity is the minimum velocity that should be given to the body to enable it to escape away form the gravitational field of earth.

b) The energy given to the body to project it with the escape velocity is called the 'Escape Energy' or 'Binding Energy'.

c) Total energy of a body is reduced to zero to enable it to escape away from the gravitational field of earth.

d) The gravitational potential energy of a particle at the surface of earth. = $\frac{-GM_e m}{R_e}$

$\therefore$ Escape Energy or Binding Energy = + $\frac{GM_e m}{R_e}$

if thrown with the velocity v_e,

then $\frac{1}{2} m v_e^2 = \frac{GM_e m}{R}$

Escape velocity for earth

$$v_e = \sqrt{\frac{2GM_e}{R_e}} = 11.2 \text{ km/sec.}$$

e) The value of escape velocity does not depend upon the mass of the projected body, instead it depends on the mass and radius of the planet from which it is being projected.

f) There is no atmosphere on the planets where the root-mean square velocities more than the escape velocity.

g) The value of escape velocity does not depend on the angle and direction of projection instead it depends on density, mass and acceleration due to gravity of the planet.

Examples based on Escape Velocity

Ex.16 A space ship is launched into a circular orbit close to earth's surface. What additional velocity has now to be imparted to the space ship in the orbit to overcome the gravitational pull ?

(Radius of earth = 6400 km, g = 9.8 m/sec^2)

(A) 3.285 km/sec (B) 32.85 m/sec^2

(C) 11.32 km/sec (D) 7.32 m/sec

Sol. (A)

The orbital velocity of space ship, $v_0 = \sqrt{\frac{GM}{r}}$

If space, ship is very near to earth's surface,

r = Radius of earth = R $\therefore$ $v_0 = \sqrt{\frac{GM}{R}}$

$= \sqrt{Rg} = \sqrt{6.4\times10^6\times9.8}$

$= 7.9195 \times 10^3$ m/sec = 7.195 km/sec

The escape velocity of space-ship

$v_e = \sqrt{2Rg} = 7.9195\sqrt{2} = 11.2$ km/sec

Additional velocity required = 11.2 – 7.9195 = 3.2805 km/sec.

Ex.17 The ratio of the radius of the earth to that of the moon is 10. The ratio of g on earth to the moon is 6. The ratio of the escape velocity from the earth's surface to that from the moon is approximately -

(A) 10 (B) 8

(C) 4 (D) 2

Sol. (B)

The escape velocity $v_e = \sqrt{2gR}$

Now, $(V_e)_{moon} = \sqrt{2gR}$

$(V_e)_{earth} = \sqrt{2\times6g\times10R}$, so $\frac{(V_e)_{earth}}{(V_e)_{moon}} = 8$

Ex.18 Acceleration due to gravity on a planet is 10 times the value on the earth. Escape velocity for the planet and the earth are V_p and V_e respectively Assuming that the radii of the planet and the earth are the same, then -

(A) $V_P = 10\,V_e$ (B) $V_P = \sqrt{10}\,V_e$

(C) $V_p = \frac{V_e}{\sqrt{10}}$ (D) $V_P = \frac{V_e}{10}$

Sol. (B)

Escape velocity $= \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$

$\therefore \frac{V_p}{V_e} = \sqrt{\frac{g_p}{g_e}\times\frac{R_e}{R_p}}$

$= \sqrt{10\times1} = \sqrt{10}$,

$V_p = \sqrt{10}\,V_e$

POINTS TO REMEMBER

1) Relation between mass of earth and its density :

$$g = \frac{GM_e}{R_e^2} = \frac{G\frac{4}{3}\pi R_e^3 d}{R_e^2}$$

$$\therefore d = \frac{3g}{4\pi GR_e} = 5.47 \text{ gm /cm}^3$$

d = density

2) Mass of Earth (M_e) :

$$M_e = \frac{gR_e^2}{G} = 6.6 \times 10^{24}\text{kg (approx.)}$$

3) Value of 'g' on other planets :

$$\frac{g}{g_e} = \left(\frac{M}{M_e}\right)\left(\frac{R_e}{R}\right)^2$$

4) Comparison of masses of two planets :

$$\frac{M_1}{M_2} = \left(\frac{T_1}{T_2}\right)^2\left(\frac{r_1}{r_2}\right)^3$$

where T_1 = Time period of I^st planet.

T_2 = Time period of II^nd planet.

r_1 = Orbital radius of I^st planet.

r_2 = Orbital radius of II^nd planet.

5) Mass of Sun :

$$M_s = \frac{4\pi^2r^3}{T^2G} = 19.72 \times 10^{29} \text{ kg}$$

where T = Time period of Earth = 365 days.

r = Distance of earth form sun.

6) If we throw a body from earth towards moon, then as body moves away from the surface of earth the value g_{earth} decreases, as a result the weight (W = mg on earth) decreases. At a particular height

$g_{earth} = g_{moon}$ therefore W = 0. Now above this height g_{moon} becomes effective and weight (W) increases till body reaches, at the surface of the moon.

7) If two mass m and M moves towards each other due to gravitational force, from rest, then relative velocity of approach will be, $\sqrt{\frac{2G(M+m)}{r}}$ where r is separation between masses.

8) Maximum height attained by a projectile (h) is given by : $h = \frac{v^2R}{2gR - v^2}$

9) If h_1 is the maximum height attained by a man on a planet whose acceleration due to gravity is g_1 and h_2 is the maximum height attained on a planet whose accleration due to gravity is g_2 then.

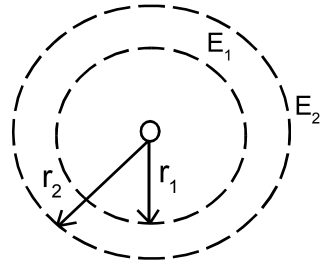
$$g_1h_1 = g_2h_2$$

10) On clock based on spring, the effect of acceleration due to gravity is negligible but on pendulum based clock effect of 'g' is noticed if 'g' increases, time period decreases and the clock moves fast if 'g' decreases time period increases and the clock moves slow.

11) Work done in changing the orbit of satellite :
W = Change in total energy

$\because$ Total $E = -\frac{GM_em}{2r}$

$$W = E_2 - E_1 = \frac{GM_em}{2}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$



12) If satellite is moved to higher orbit then potential energy, angular momentum, increases but kinetic energy, binding energy, orbital velocity, decrease.

SOLVED EXAMPLES

Ex.1 The Jupiter's period of revolution round the sun is 12 times that of the earth. Assuming the planetary orbits to circular, how many times the distance between the jupiter and sun exceeds that between the earth and the sun.

(A) 5.242 (B) 4.242
(C) 3.242 (D) 2.242

Sol **(A)**

We know that $T^2 \propto a^3$

Given that $(12\ T)^2 \propto a_1^3$ and $T^2 \propto a_2^3$

$$\therefore \frac{a_1^3}{a_2^3} = \frac{(12T)^2}{T^2} = 144$$

or $\frac{a_1}{a_2} = (144)^{1/3} = 5.242$

Hence the jupiter's distance is 5.242 times that of the earth from the sun.]

Ex.2 The mean distance of mars from sun is 1.524 times the distance of the earth from sun. The period of revolution of mars around sun will be-

(A) 2.88 earth year (B) 1.88 earth year
(C) 3.88 earth year (D) 4.88 earth year

Sol **(B)**

We know that $T^2 \propto a^3 \Rightarrow T \propto (a)^{3/2}$

$$\therefore \frac{T_{mars}}{T_{earth}} = \left(\frac{a_{mars}}{a_{earth}}\right)^{\frac{3}{2}} = (1.524)^{3/2} = 1.88$$

As the earth revolves round the sun in one year and hence,

$T_{earth} = 1$ year.

$\therefore$ $T_{mars} = T_{earth} \times 1.88 = 1 \times 1.88$

$= 1.88$ earth-year.

Ex.3 The semi-major axes of the orbits of Mercury and Mars are respectively 0.387 and 1.524 in astronomical units. If the period of Mercury is 0.241 year, what is the period of Mars.

(A) 1.2 years (B) 3.2 years
(C) 3.9 years (D) 1.9 years

Sol **(D)**

$$\frac{T_{mercury}}{T_{mars}} = \left(\frac{a_{mercury}}{a_{mars}}\right)^{3/2} = \left(\frac{0.387}{1.524}\right)^{3/2}$$

$$\therefore T_{mars} = T_{mercury} \times \left(\frac{1.524}{0.387}\right)^{3/2}$$

$= (0.241\text{years}) \times (7.8) = 1.9$ years.

Ex.4 If a graph is plotted between T^2 and r^3 for a planet then its slope will be -

(A) $\frac{4\pi^2}{GM}$ (B) $\frac{GM}{4\pi^2}$

(C) 4π GM (D) 0

Sol **(A)**

$$\frac{T^2}{r^3} = \frac{\left(\frac{2\pi r}{v_0}\right)^2}{r^3} = \frac{(2\pi r)^2}{r^3}\ \frac{1}{GM}\ r = \frac{4\pi^2}{GM}$$

$$\left[\because \frac{mv_0^2}{r} = \frac{GMm}{r^2},\ v_0^2 = \frac{GM}{r}\right]$$

Slope of $T^2 - r^3$ curve $= \tan\theta$

$$= \frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$

Ex.5 Four particles, each of mass m, are placed at the corners of squar and moving along a circle of radius r under the influence of mutual gravitational attraction. The speed of each particle will be -

(A) $\sqrt{\frac{Gm}{r}}\ (2\sqrt{2} + 1)$ (B) $\sqrt{\frac{Gm}{r}}$

(C) $\sqrt{\frac{Gm}{r}\left(\frac{2\sqrt{2}+1}{4}\right)}$ (D) $\sqrt{\frac{2\sqrt{2}Gm}{r}}$

Sol **(C)**

Resultant force on particle '1'

$$F_r = \sqrt{2}\ F + F'$$

or $$F_r = \sqrt{2}\ \frac{Gm^2}{2r^2} + \frac{Gm^2}{4r^2} = \frac{mv^2}{r}$$

or $$v = \sqrt{\frac{Gm}{r}\left(\frac{2\sqrt{2}+1}{4}\right)}$$

Ex.6 Three particles of equal mass m are situated at the vertices of an equilateral triangle of side ℓ. What should be the velocity of each particle, so that they move on a circular path without changing ℓ -

(A) $\sqrt{\frac{GM}{2\ell}}$ (B) $\sqrt{\frac{GM}{\ell}}$

(C) $\sqrt{\frac{2GM}{\ell}}$ (D) $\sqrt{\frac{GM}{3\ell}}$

Sol **(B)**

The resultant gravitational force on each particle provides it the necessary centripetal force

$$\therefore \quad \frac{mv^2}{r} = \sqrt{F^2 + F^2 + 2F^2 \cos 60^o} = \sqrt{3}\,F,$$

But. $r = \frac{\sqrt{3}}{2}\,\ell \times \frac{2}{3} = \frac{\ell}{\sqrt{3}}$,

$$\therefore \quad v = \sqrt{\frac{GM}{\ell}}$$

Ex.7 What will be the acceleration due to gravity on the surface of the moon if its radius is 1/4 th the radius of the earth and its mass is 1/80 th the mass of the earth.

(A) g/6 (B) g/5
(C) g/7 (D) g/8

Sol **(B)**

The acceleration due to gravity on the surface of the earth, in terms of mass M_e and radius R_e of earth, is given by

$$g = \frac{GM_e}{R_e^{\,2}}$$

if M_m be the mass of the moon, R_m its radius, then the acceleration due to gravity on the surface of the moon will be given by

$$g' = \frac{GM_m}{R_m^{\,2}}$$

Dividing eq. (ii) by eq. (i), we get

$$\frac{g'}{g} = \frac{M_m}{M_e}\left(\frac{R_e}{R_m}\right)^2 = \frac{1}{80} \times \left(\frac{4}{1}\right)^2 = \frac{1}{5}$$

$\therefore$ $g' = g/5.$

Ex.8 The speed with which the earth would have to rotate on its axis so that a person on the equator would weight 3/5 th as much as at present, will be (Take the equatorial radius as 6400 km.)

(A) 7.8×10^{-3} radian/sec.
(B) 7.8×10^{-4} radian/sec.
(C) 7.8×10^{-5} radian/sec.
(D) 7.8×10^{-2} radian/sec.

Sol **(B)**

At present the weight of the person on the equator is nearly the same which would have been if the earth were stationary. Suppose, for the weight to remain 3/5 th, the angular speed of earth is ω. Then according to the formula $g' = g - R_e\,\omega$, we have

$$\frac{3}{5}mg = mg - m\,R_e\,\omega^2.$$

$$\therefore \quad \omega = \sqrt{\left(\frac{2g}{5R_e}\right)} = \sqrt{\left(\frac{2 \times 9.8 m/s^2}{5 \times (6400 \times 10^3 m)}\right)}$$

$= 7.8 \times 10^{-4}$ radian/sec.

Ex.9 At what height above the earth's surface the acceleration due to gravity will be 1/9 th of its value at the earth's surface ? Radius of earth is 6400 km.

(A) 12800km (B) 1280km
(C) 128000km (D) 128km

Sol **(A)**

If g be the acceleration due to gravity at the surface of the earth, then its value at a height h above the earth's surface will be -

$$g' = \frac{g}{\left(1 + \frac{h}{R_e}\right)^2}$$

Here $\frac{g'}{g} = \frac{1}{9} \quad \therefore \quad \frac{1}{9} = \frac{1}{\left(1 + \frac{h}{R_e}\right)^2}$

or $1 + \frac{h}{R_e} = 3$

or $h = 2\,R_e = 2 \times 6400 = 12800$ km.

Ex.10 If the radius of the earth were to shrink by one percent, its mass remaining the same, the acceleration due to gravity on the earth's surface would -

(A) decrease,
(B) remain unchanged,
(C) increase.
(D) None of these

Sol **(C)**

Consider the case of a body of mass m placed on the earth's surface (mass of the earth M and radius R). If g is acceleration due to gravity, then

$$mg = G\,\frac{M_e m}{R^2} \text{ or } g = \frac{GM_e}{R^2}$$

where G is universal constant of gravitation. Now when the radius is reduced by 1%, i.e., radius becomes 0.99 R, let acceleration due to gravity be g', then

$$g' = \frac{GM_e}{(0.99R)^2}$$

From equation (A) and (B), we get

$$\frac{g'}{g} = \frac{R^2}{(0.99R)^2} = \frac{1}{(0.99)^2}$$

$$\therefore \quad g' = g \times \left(\frac{1}{0.99}\right)^2 \text{ or } g' > g$$

Thus , the value of g is increased.

Ex.11 At what height above the Earth's surface does the force of gravity decrease by 10%. Assume radius of earth to be 6370 km.

(A) 350km. (B) 250km.
(C) 150km. (D) 300km.

Sol **(A)**

Force of gravity at surface of earth,

$F_1 = Gm\ M/R^2$ --------------------(1)

Force of gravity at height H is

$F_2 = Gm\ M\ (R + H)^2$ --------------------(2)

Dividing (A) by (B) and Rearranging

$$H = R\left(\sqrt{\frac{F_1}{F_2}} - 1\right) = 350 \text{ km where } (F_2 = .9F_1)$$

Ex.12 A particle is suspended from a spring and it stetches the spring by 1 cm on the surface of earth. The same particle will stretches the same spring at a place 800 km above earth surface by -

(A) 0.79 cm (B) 0.1 cm
(C) $\pi/6$ rad/hr (D) $2\pi/7$ rad/hr

Sol (A)

The extension in the length of spring is

$$x = \frac{mg}{k} = \frac{GMm}{r^2k},$$

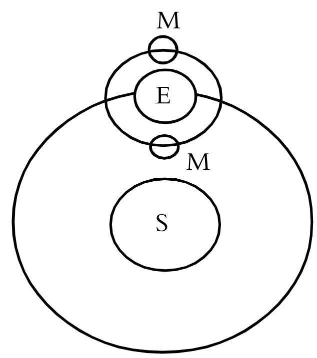
$$\therefore \ x \propto \frac{1}{r^2}, \ \therefore \ \frac{x_2}{x_1} = \frac{R^2}{(R+h)^2}$$

$$\text{or} \quad x_2 = 1 \times \left(\frac{6400}{7200}\right)^2 = 0.79 \text{ cm}.$$

Ex.13 The change in the value of acceleration of earth towards sun, when the moon comes from the position of solar eclipse to the position on the other side of earth in line with sun is - (mass of moon = 7.36×10^{22} kg, the orbital radius of moon 3.8×10^8m.

(A) 6.73×10^{-2} m/s^2
(B) 6.73×10^{-3} m/s^2
(C) 6.73×10^{-4} m/s^2
(D) 6.73×10^{-5} m/s^2

Sol **(D)**



In the position of solar eclipse, net force on earth $F_E = F_M + F_S$

In the position of lunar eclipse, net force on earth $F_E' = F_S - F_M$

$\therefore$ Change in acceleration of earth,

$$\Delta f = \frac{2GM}{R^2} = \frac{2\times 6.67\times 10^{-11}\times 7.36\times 10^{22}}{3.82^2\times 10^{16}}$$

$$= 6.73 \times 10^{-5} \text{ m/s}^2$$

Ex.14 With what velocity must a body be thrown upward form the surface of the earth so that it reaches a height of 10 R_e? Earth's mass $M_e = 6 \times 10^{24}$ kg, radius $R_e = 6.4 \times 10^6$ m and $G = 6.67 \times 10^{-11}$ N-m^2/kg^2.

(A) 10.7×10^4 m/s
(B) 10.7×10^3 m/s
(C) 10.7×10^5 m/s
(D) 1.07×10^4 m/s

Sol **(D)**

Let m be the mass of the body. The gravitational potential energy of the body at the surface of the earth is

$$U = -\frac{GM_e m}{R_e}$$

The potential energy at a height 10 R_e above the surface of the earth will be

$$U' = -\frac{GM_e m}{(R_e + 10R_e)}$$

$\therefore$ increase in potential energy

$$U' - U = -\frac{GM_e m}{11R_e} + \left(\frac{GM_e m}{R_e}\right)$$

$$= \frac{10}{11}\frac{GM_e m}{R_e}$$

This increase will be obtained from the initial kinetic energy given to the body. Hence if the body be thrown with a υ velocity then

$$\frac{1}{2} m\upsilon^2 = \frac{10}{11}\frac{GM_e m}{R_e}$$

$$\Rightarrow \quad \upsilon = \sqrt{\frac{20Gm_e}{11R_e}}$$

Substituting the given values, we get

$$\upsilon = \sqrt{\left(\frac{20\times(6.67\times10^{-11})\times(6\times10^{24})}{11\times(6.4\times10^{6})}\right)}$$

$$= 1.07 \times 10^4 \text{ m/s.}$$

Ex.15 The radius of the earth is R_e and the acceleration due to gravity at its surface is g. The work required in raising a body of mass m to a height h form the surface of the earth will be -

(A) $\dfrac{mgh}{\left(1-\dfrac{h}{R_e}\right)}$ (B) $\dfrac{mgh}{\left(1+\dfrac{h}{R_e}\right)^2}$

(C) $\dfrac{mgh}{\left(1+\dfrac{h}{R_e}\right)}$ (D) $\dfrac{mg}{\left(1+\dfrac{h}{R_e}\right)}$

Sol (C)

Let M_e be the mass of the earth. The work required

$$W = GM_e\, m\left[\frac{1}{R_e} - \frac{1}{R_e + h}\right]$$

$$= \frac{GM_e mh}{R_e(R_e+h)} = \frac{gR_e^2 mh}{R_e(R_e+h)}$$

$$[\therefore GM_e = gR_e^2] = \frac{mgh}{\left(1+\dfrac{h}{R_e}\right)}$$

Ex.16 If the satellite is stopped suddenly in its orbit which is at a distnace = radius of earth from earth's surface and allowed to fall freely into the earth, the speed with which it hits the surface of earth will be -

(A) 7.919 m/sec (B) 7.919 km/sec

(C) 11.2 m/sec (D) 11.2 km/sec

Sol **(B)**

From conservation of energy,

The energy at height h = Total energy at earth's surface

$$0 - \frac{GMm}{R+h} = \frac{1}{2} mv^2 - \frac{GMm}{R},$$

$$\frac{1}{2} mv^2 = \frac{GMm}{R} - \frac{GMm}{R+h} = \frac{GMm}{R} - \frac{GMm}{2R}$$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{R}} = \sqrt{\frac{R^2 g}{R}} = \sqrt{Rg}$$

$$= \sqrt{6400\times10^3\times9.8} = 7.919 \times 10^3 \text{ m/s}$$

$= 7.919$ km/sec .

Ex.17 A projectile is fired vertically upward from the surface of earth with a velocity K v_e, where v_e is the escape velocity and K < 1. Neglecting air resistance, the maximum height to which it will rise measured from the centre of the earth is - (where are R = radius of earth)

(A) $\dfrac{R}{1-K^2}$ (B) $\dfrac{R}{K^2}$

(C) $\dfrac{1-K^2}{R}$ (D) $\dfrac{K^2}{R}$

Sol **(A)**

If a body is projected from the surface of earth with a velocity v and reaches a height h, then using law of conservation of energy,

$\frac{1}{2} mv^2 = \frac{mgh}{1+h/R}$. Given $v = Kv_e = K\sqrt{2gR}$

and $h = r - R$

Hence, $\dfrac{1}{2} mK^2\, 2gR = \dfrac{mg(r-R)}{1+\dfrac{r-R}{R}}$

or $r = \dfrac{R}{1-K^2}$

Ex.18 A satellite is revolving in an orbit close to the earth's surface. Taking the radius of the earth as 6.4×10^6 meter, the value of the orbital speed and the period of revolution of the satellite will respectively be. (g = 9.8 meter/sec^2)

(A) 7.2 km/sec., 84.6 minutes
(B) 2.7 km/sec., 8.6 minutes
(C) .72 km/sec., 84.6 minutes
(D) 7.2 km/sec., 8.6 minutes

Sol Orbital speed,

$$\upsilon_o = \sqrt{g R_e} = \sqrt{9.8 \times (6.4 \times 10^6)}$$

$= 7.2 \times 10^3$ m/s = **7.2 km/s**.

Period of revolution, $T = 2\pi \sqrt{R/g}$

$$= 2 \times 3.14 \sqrt{(6.4 \times 10^6)/9.8}.$$

= 5075 s = 84.6 minutes.

Ex.19 If the period of revolution of an artificial satellite just above the earth be T and the density of earth be ρ, then

(A) ρT^2 is a universal constant.
(B) ρT^2 varies with time
(C) $\rho T^2 = \dfrac{3\pi}{G}$
(D) $\rho T^2 = 3\pi \times G$

Sol **(A), (C)**

If the period of revolution of a satellite about the earth be T, then

$$T^2 = \frac{4\pi^2 (R_e + h)^3}{GM_e}$$

where h is the height of the satellite from earth's surface.

$$\therefore \quad M_e = \frac{4\pi^2 (R_e + h)^3}{GT^2}$$

The satellite is revolving just above the earth, hence h is negligible compared to R_e.

$$\therefore \quad M_e = \frac{4\pi^2 R_e^3}{GT^2}$$

But $M_e = \frac{4}{3}\pi R_e^3 \rho$ where ρ is the density of the earth. Thus

$$\frac{4}{3}\pi R_e^3 \rho = \frac{4\pi^2 R_e^3}{GT^2}$$

$$\therefore \quad \rho T^2 = \frac{3\pi}{G}.$$

which is universal constant. To determine its value,

$$\rho T^2 = \frac{3\pi}{G} = \frac{3 \times 3.14}{6.67 \times 10^{-11} \text{ m}^3/\text{kg} - \text{s}^2}$$

Ex.20 A body of mass 100 kg falls on the earth from infinity. What will be its energy on reaching the earth ? Radius of the earth is 6400 km and g = 9.8 m/s^2. Air friction is negligible.

(A) 6.27×10^9 J
(B) 6.27×10^{10} J
(C) 6.27×10^{10} J
(D) 6.27×10^7 J

Sol **(A)**

A body projected up with the escape velocity υ_e will go to infinity. Therefore, the velocity of the body falling on the earth from infinity will be υ_e. Now, the escape velocity on the earth is

$$\upsilon_e = \sqrt{gR_e}$$

$$= \sqrt{2 \times (9.8 \text{m/s}^2) \times (6400 \times 10^3 \text{m})}$$

$= 1.2 \times 10\ 10^4$ m/s = 11.2 km/s.

The kinetic energy acquired by the body is

$$K = \frac{1}{2} m \upsilon_e^2$$

$$= \frac{1}{2} \times 100 \text{ kg} \times (11.2 \times 10^3 \text{ m/s})^2$$

$= 6.27 \times 10^9$ J.

Ex.21 An artificial satellite of the earth is to be established in the equatorial plane of the earth and to an observer at the equator it is required that the satellite will move eastward, completing one round trip per day. The distance of the satellite from the centre of the earth will be- (The mass of the earth is 6.00×10^{24} kg and its (angular velocity = 7.30×10^{-5} rad./sec.)

(A) 2.66×10^3 m.
(B) 2.66×10^5 m.
(C) 2.66×10^6 m.
(D) 2.66×10^7 m.

Sol **(D)**

We know that

$$\frac{GMm}{r^2} = m\,\omega^2 r \text{ or } \frac{GM}{r^2} = \omega^2 r.$$

$$\therefore \quad r^3 = \frac{GM}{\omega^2}$$

where ω is the angular velocity of the satellite

In the present case, $\omega = 2\omega_0$,

where ω_0 is the angular velocity of the earth.

$\therefore \quad \omega = 2 \times 7.3 \times 10^{-5}$ rad/ sec.

$G = 6.673 \times 10^{-11}$ n-m^2/kg^2

and $M = 6.00 \times 10^{24}$kg.

Substituting these values in equation (A), we get

$$r^3 = \frac{(6.673\times10^{-11})(6.00\times10^{24})}{(2\times7.3\times10^{-5})^2}$$

Solving we get $r = 2.66 \times 10^7$m.

Ex.22 Two satellites P and Q of same mass are revolving near the earth surface in the equitorial plane. The satellite P moves in the direction of rotation of earth whereas Q moves in the apposite direction. The ratio of their kinetic energies with respect to a frame attached to earth will be -

(A) $\left(\frac{8363}{7437}\right)^2$ (B) $\left(\frac{7437}{8363}\right)^2$

(C) $\left(\frac{8363}{7437}\right)$ (D) $\left(\frac{7437}{8363}\right)$

Sol (A)

$\frac{E_{KQ}}{E_{KP}} = \frac{v_Q^2}{v_P^2}$. Linear velocity of earth,

$$V_e = \frac{2\pi R_e}{T_e} = \frac{6.28\times6.4\times10^6}{24\times3600} = 463 \text{ m/s}$$

Orbital velocity, $V_o = \sqrt{R_e g} = 7.9 \times 10^3$ m/s

According to question,

$V_P = V_0 + V_e = 7900 - 463 = 7437$ m/s

$V_Q = V_o + V_e = 7900 + 463 = 8363$ m/s

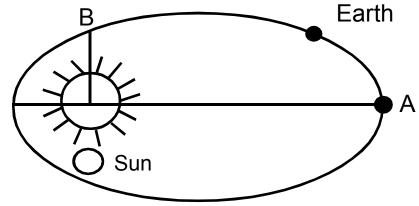
$$\therefore \frac{E_{KQ}}{E_{KP}} = \left(\frac{8363}{7437}\right)^2$$

Ex.23 The mass of the sun if the mean radius of earth's orbit is 1.5×10^8 km and $G = 6.67 \times 10^{-11}$ Nm^2 /kg^2 will be-

(A) 2×10^{10} kg (B) 3×10^{10} kg
(C) 2×10^{30} kg (D) 3×10^{30} kg

Sol **(C)**

We know that, $v = \sqrt{\frac{GM}{r}}$,



$$\therefore \quad T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{GM}}$$

or $M = \frac{4\pi^2 r^3}{GT^2}$

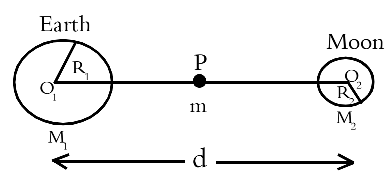
(As $T = 1$ year $= 3.15 \times 10^7$ sec)

Ex.24 The masses and the radius of the earth and the moon are M_1, M_2 and R_1, R_2 respectively their centres are at distance d apart. The minimum speed with which a particle of mass m should be projected form a point midway between the two centres so as to escape to infinity will be -

(A) $2\sqrt{\frac{G}{d}(M_1+M_2)}$ (B) $\sqrt{\frac{G}{d}(M_1+M_2)}$

(C) $\sqrt{\frac{G}{2d}(M_1+M_2)}$ (D) $2\sqrt{\frac{G}{d}\frac{M_1}{M_2}}$

Sol (A) The P.E of the mass at d/2 due to the earth and moon is



$$U = -2\frac{GM_1m}{d} - 2\frac{GM_2m}{d}$$

or $U = -\frac{2Gm}{d}(M_1 + M_2)$

(Numerically)

$$\frac{1}{2} m V_e^2 = U$$

$$\Rightarrow \quad V_e = 2\sqrt{\frac{G}{d}(M_1+M_2)}$$

Ex.25 The diameter of a planet is four times that of the earth. The time period of a pendulum on the planet, if it is a second pendulum on the earth will be- (Take the mean density of the planet equal to that of the earth).

(A) 1s (B) 2s

(C) 3s (D) 4s

Sol **(A)**

If R be the radius of the earth, then radius of the planet is 4R

Acceleration due to gravity on earth

$$g = \frac{GM}{R^2} = G\,\frac{4}{3}\,\frac{\pi R^3 \rho}{R_e^{\,2}} = \frac{4\pi R\rho}{3}\,G.$$

Acceleration due to gravity on planet,

$$g' = \frac{GM'}{(4R)^2} = \frac{G4\pi(4R)^3\rho}{3(4R)^2} = \frac{16\pi R\rho}{3}\;G.$$

Time period of the pendulum on earth.

$$T = 2 = 2\pi\sqrt{\frac{\ell}{g}}\ \text{sec}$$

T' be the time period of the pendulum at planet, then

$$T' = 2\pi\sqrt{\frac{\ell}{g'}} = 2\pi\sqrt{\frac{\ell}{4g}} = \pi\sqrt{\frac{\ell}{g}},$$

$$\therefore\ T'/T = \frac{1}{2} \text{ or } T' = 2 \times \frac{1}{2} = 1 \text{ sec.}$$