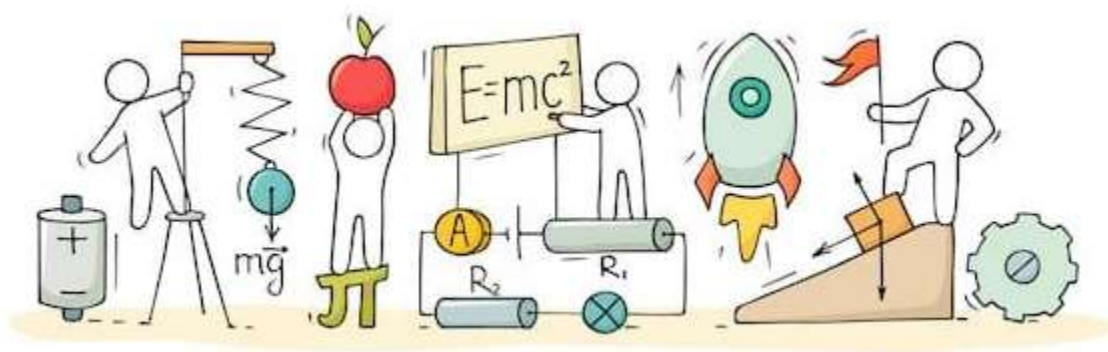


PHYSICS



ELECTROSTATIC POTENTIAL AND CAPACITANCE

Introduction:

In the previous chapter, we have learnt about "Electric Charges and Fields". In this chapter, we shall focus Electrostatic Potential and Capacitance. The energy point of view can be used in electricity, and it is especially useful. Energy is also a tool in solving Problems more easily in many cases than by using forces and electric fields.

Electric energy can be stored in a common device called a capacitor, which is found in nearly all electronic circuits. A capacitor is used as a storehouse for energy. Capacitors store the energy in common photo flash units.

Electrostatic Potential:

The electrostatic potential (V) at any point in a region with electrostatic field is the work done in bringing a unit positive charge (without acceleration) from infinity to that point. If 'W' is the work done in moving a charge 'q' from infinity to a point, then the potential at that point is $V = \frac{W}{q}$

Electric Potential Difference:

Similar to electric potential, the electric potential difference is the work done by external force in bringing a unit positive charge from point R to point P. i.e.,

$$V_P - V_R = \frac{U_P - U_R}{q}$$

Here V_P and V_R are the electrostatic potentials at P and R, respectively and U_P and U_R are the potential energies of a charge q when it is at P and at R respectively.

Note: As before, that it is not the actual value of potential but the potential difference that is physically significant. If, as before, we choose the potential to be zero at infinity, the above equation implies.

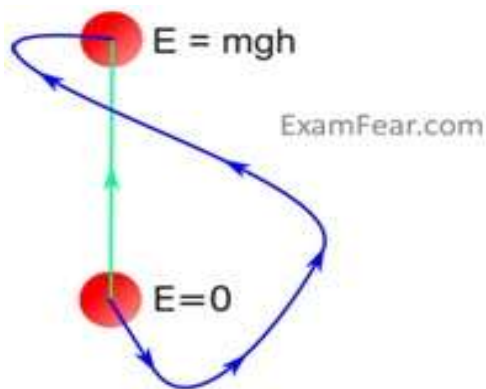
Unit for Electric Potential:

The unit of measurement for electric potential is the volt, so electric potential is often called voltage. A potential of 1 volt (V) equals 1 joule (J) of energy per 1 coulomb (C) of charge.

$$1V = 1 \frac{J}{C}$$

Conservative Forces:

When one form of energy gets converted to another completely on application or removal of external force, the forces are said to be conservative. Examples of conservative forces are sum of kinetic and potential energies working on a body, spring and gravitational force, coulomb force between two stationary charges, etc.

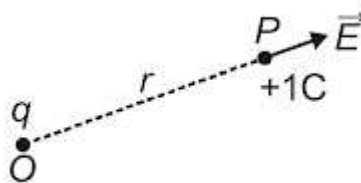


Work done by conservative gravitational force is same for different paths followed by a particle to reach from one point to another.

Work done in moving an object from one point to another depends only on the initial and final positions and is independent of the path taken.

Potential due to a Point Charge:

Consider a point charge q placed at point O . Consider any point P in the field of the above charge. Let us calculate the potential at point P due to the charge q kept a point O . Since work done is independent of path, we choose a convenient path, along the radial direction.



Let the distance $OP = r$.

The electric force at P , due to q will be directed along OP , given by

$$F = \frac{1}{4\pi\epsilon_0} = \frac{qq_0}{r^2}$$

If the work done by moving this positive charge to dr distance is dW then,

$$dW = F (-dr)$$

$$dW = - \int F \cdot dr$$

$$dW = - \int_{\infty}^r F. dr$$

Hence, the total work done in bringing this charge from (∞) to 'r' will be,

$$W = \int_{\infty}^r \frac{1}{4\pi\epsilon_0} \frac{qq_0}{r^2} . dr$$

$$W = -\frac{qq_0}{4\pi\epsilon_0} \int_{\infty}^r \frac{1}{r^2} . dr$$

$$W = -\frac{qq_0}{4\pi\epsilon_0} \left[-\frac{1}{r} \right]_{\infty}^r$$

$$W = \frac{1}{4\pi\epsilon_0} \frac{qq_0}{r^2}$$

Hence, from $V = \frac{W}{q_0}$ electric potential is,

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

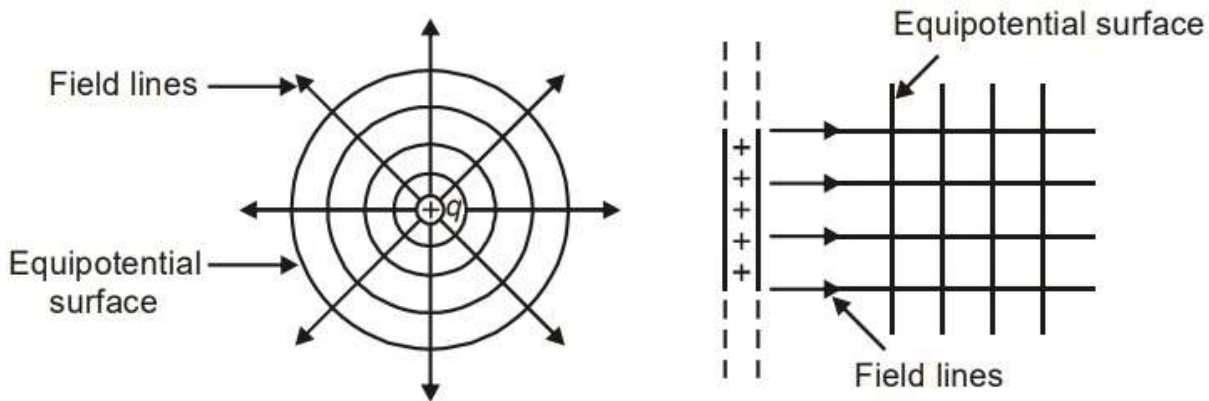
This equation is true for any sign of charge q . For $q < 0$, $V < 0$, i.e., work done by the external force per unit positive test charge to bring it from infinity to the point is negative. Also, this equation is consistent with the choice that potential at infinity be zero.

Equipotential Surfaces:

An equipotential surface is a surface with a constant value of potential at all points on the surface. For a single charge q , the potential is given by

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

This shows that V is a constant if r is constant. Thus, equipotential surfaces of a single point charge are concentric spherical surfaces centered at the charge.

**Example:**

- Surface of a charged conductor.
- All points equidistant from a point charge.

Note:

- An equipotential surface is that at which, every point is at the same potential. As the work done is given by $(V_A - V_B)q_0$.
- Work done by electric field while a charge moves on an equipotential surface is zero as $V_A = V_B$.

Electrostatics of Conductors:

Conductors contain mobile charge carriers. In metallic conductors, these charge carriers are electrons. In a metal, the outer (valence) electrons part away from their atoms and are free to move. These electrons are free within the metal but not free to leave the metal.

Whenever a conductor is placed in an external electric field, the free electrons in it experience a force due to it and start moving opposite to the field. This movement makes one side of conductor positively charged and the other as negatively charged. This creates an electric field in the conductor in a direction opposite to external electric field (called induced field).

Important Points about Electrostatics of Conductors:

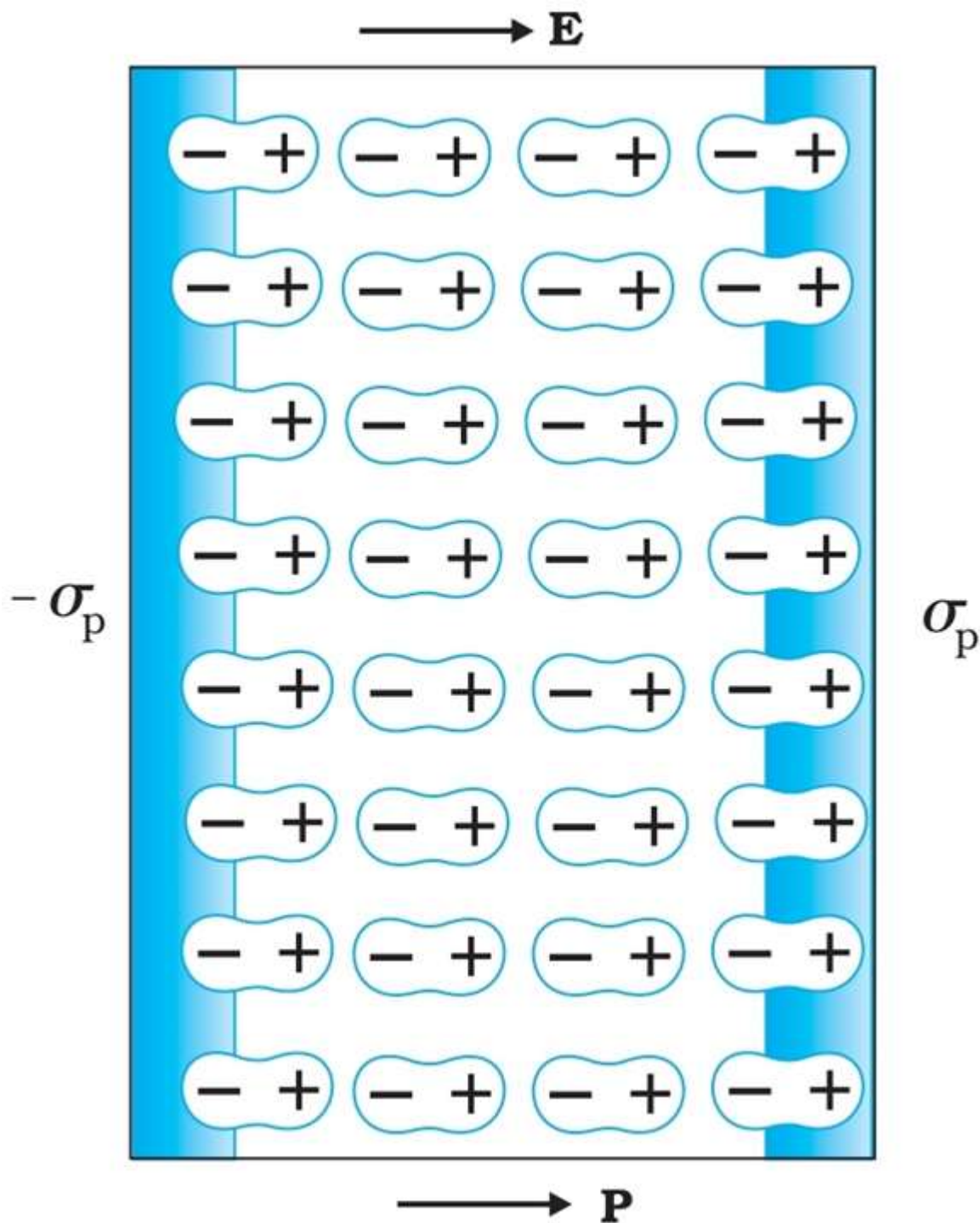
- **Inside a conductor, electrostatic field is zero:** In the previous chapter, we have already discussed that “when there is no electric current inside or on the surface of a conductor, the electric field inside the conductor is everywhere zero”.
- **At the surface of a charged conductor, electrostatic field must be normal to the surface at every point:** If the field E is not normal to the surface, it will have a nonzero component along the surface. Hence the free charge on the surface will move due to electrostatic force on it. But free charge on the surface in electrostatics remains at rest. So, the electrostatic

field at the surface of a charged conductor must be normal to the surface.

- **Electrostatic Shielding:** In an electrostatic situation, if a conductor contains a cavity and if no charge is present inside the cavity, then there can be no net charge anywhere on the surface of the cavity. This means that if you are inside a charged conducting box, you can safely touch any point on the inside walls of the box without being electrocuted. This is known as electrostatic shielding.

Dielectrics and Polarization:

Dielectrics are non-conducting substances. In contrast to conductors, they have no (or negligible number of) charge carriers. When a conductor is placed in an external electric field, the free charge carriers move and charge distribution in the conductor adjusts itself in such a way that the electric field due to induced charges opposes the external field within the conductor. This happens until, in the static situation, the two fields cancel each other and the net electrostatic field in the conductor is zero.

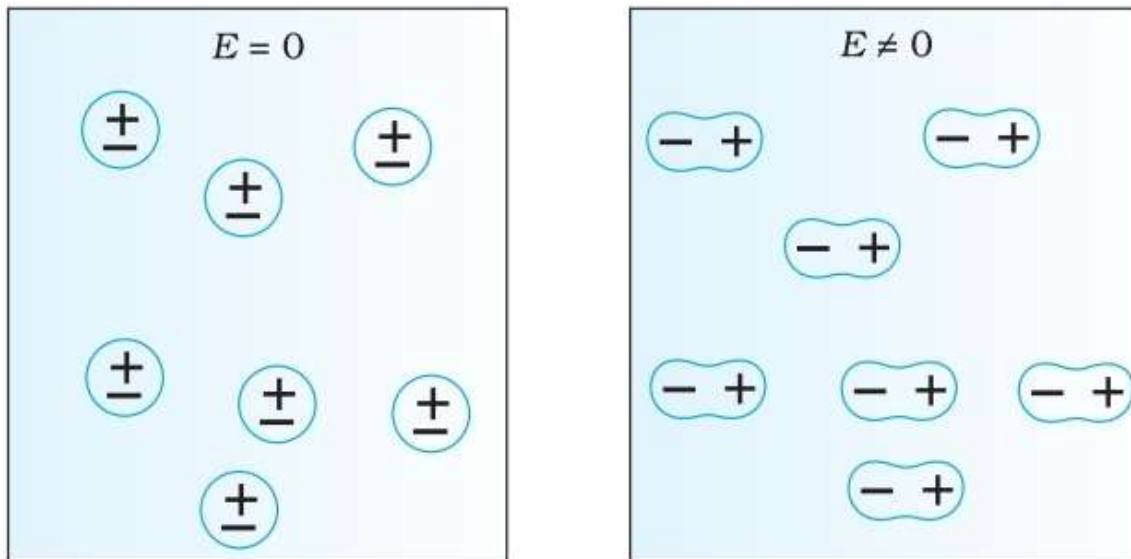


When a dielectric material is kept in an electric field, the external field induces dipole moment by stretching or reorienting molecules of the dielectric. This results in development of net charges on the surface of the dielectric which produce a field that opposes the external field.

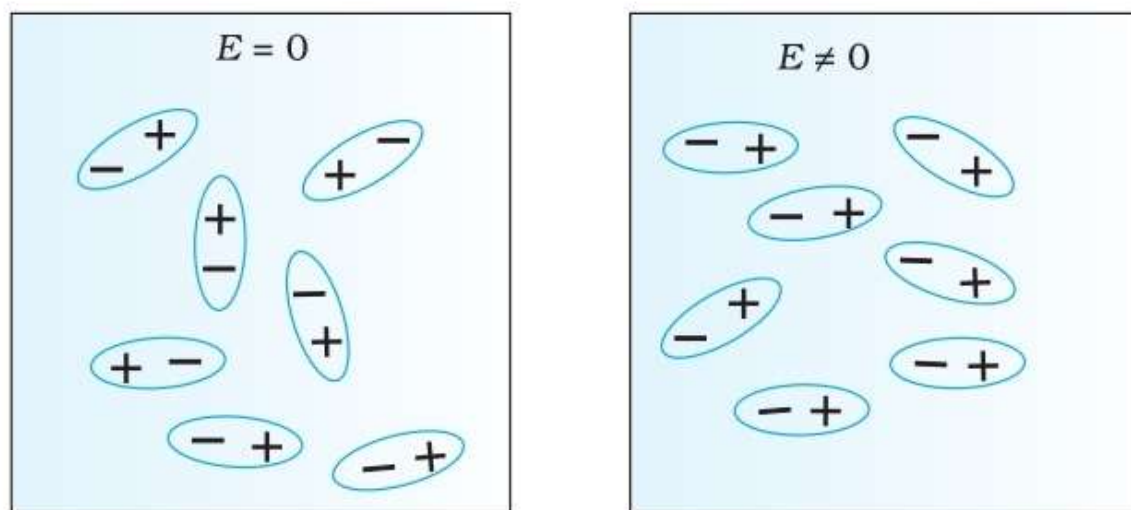
In general, the dielectric can be classified into Polar and Non-polar dielectrics. In a non-polar molecule, the centers of positive and negative charges coincide. The molecule thus has no permanent dipole moment. Examples of non-polar molecules are oxygen (O_2) and hydrogen (H_2) molecules which, because of their symmetry, have no dipole moment. On the other hand, a polar molecule is one in which the centers of positive and negative charges are separated (even

when there is no external field). Such molecules have a permanent dipole moment. An ionic molecule such as HCl or a molecule of water (H_2O) are examples of polar molecules.

Behavior of a non-polar dielectric: In an external electric field, the positive and negative charges of a nonpolar molecule are displaced in opposite directions. The displacement stops when the external force on the constituent charges of the molecule is balanced by the restoring force. The non-polar molecule thus develops an induced dipole moment. The dielectric is said to be polarized by the external field.



(a) Non-polar molecules



(b) Polar molecules

Behavior of a polar dielectric: A dielectric with polar molecules also develops a net dipole moment in an external field, but for a different reason. In the absence of any external field, the different permanent dipoles are oriented randomly due to thermal agitation; so, the total dipole moment is zero. When an external field is applied, the individual dipole moments tend to align

with the field.

Capacitors and Capacitance:

A capacitor is a system of two conductors separated by an insulator. If two conductors have a potential difference between them then, as any potential difference is able to accelerate charges, the system effectively stores energy. Such a device that can maintain a potential difference, storing energy by storing charge is called capacitor. When charges $+Q$ and $-Q$ are given to two plates, a potential difference is developed between the plates. The capacitance of the arrangement is defined as.

$$C = \frac{Q}{V}$$

Definition of Capacitance: Capacitance is defined as the amount of charge required to raise the potential of a conductor by one volt.

Capacity of an isolated spherical conductor:

Consider a sphere with center O and radius r , which is supplied with a charge $= +q$. This charge is distributed uniformly over the outer surface of the sphere. Thus, the potential at every point on the surface is same and is given by.

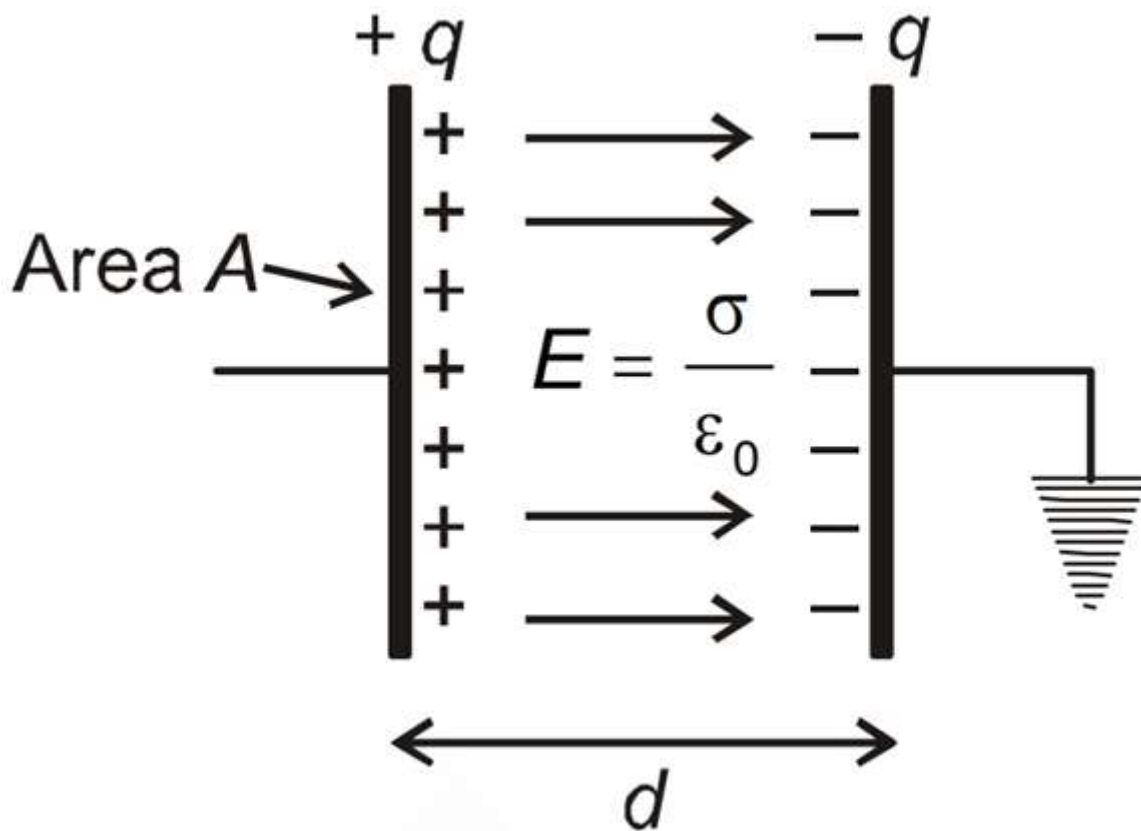
$$V = \frac{q}{4\pi\epsilon_0 r}$$

$$\text{As } C = \frac{Q}{V}$$

$$C = 4\pi\epsilon_0 r$$

The Parallel Plate Capacitor:

The arrangement consists of two thin conducting plates, each of area A and separated by a small distance d . When charge q is given to first plate, a charge $-q$ is induced on the inner face of other plate and positive on the outer face of plate. As this face is connected to earth, a net negative charge is left on this plate. Thus, the arrangement is equivalent to two thin sheets of charge. As d is much smaller than the linear dimension of the plates ($d^2 \ll A$), we can use the result of electric field by an infinite sheet of charge. The electric field between the plates is.



$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0}$$

$$E = \frac{\sigma}{\epsilon_0} \dots (1)$$

For uniform field potential difference between the plates.

$$V = Ed = \frac{\sigma d}{\epsilon_0} \dots \text{From eq (1)}$$

$$V = \frac{qd}{\epsilon_0 A} \text{ as } \sigma = \frac{q}{A}$$

$$C = \frac{q}{V} = \frac{qA}{\frac{qd}{\epsilon_0 A}}$$

$$C = \frac{\epsilon_0 A}{d}$$

Effect of Dielectric on Capacitance:

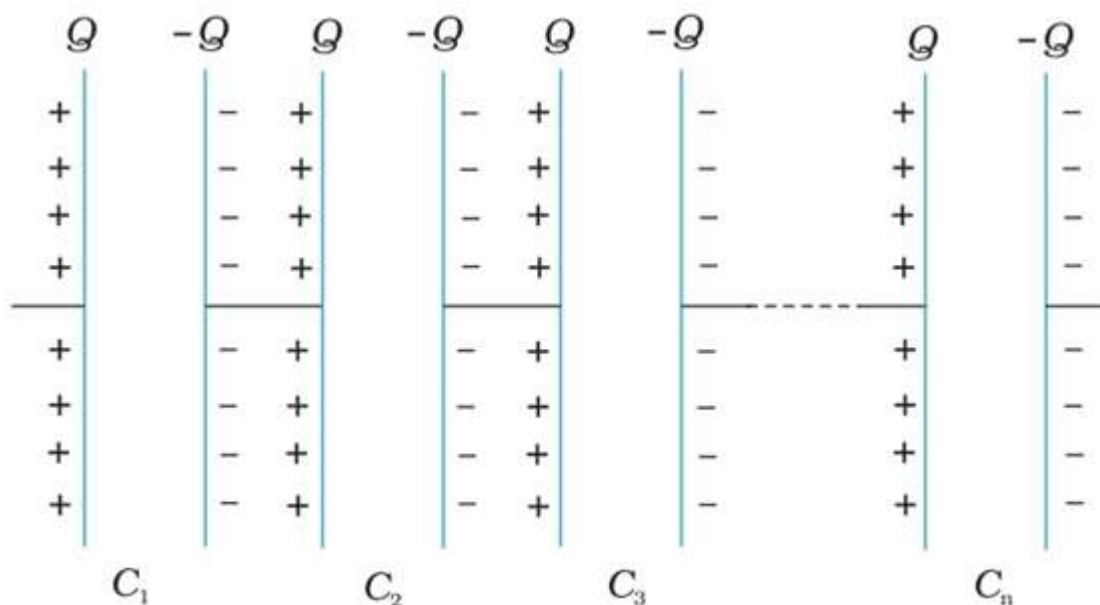
When a dielectric slab of dielectric constant K is inserted between the plates filling the entire space between the plates. The plates of the capacitor are given charge $+Q$ and $-Q$ and hence

induced charges $-QP$ and $+QP$ appear on the surfaces of the slab. So, capacitance is increased to K times when the space between the plates is filled with a dielectric of dielectric constant K .

Combination of Capacitors:

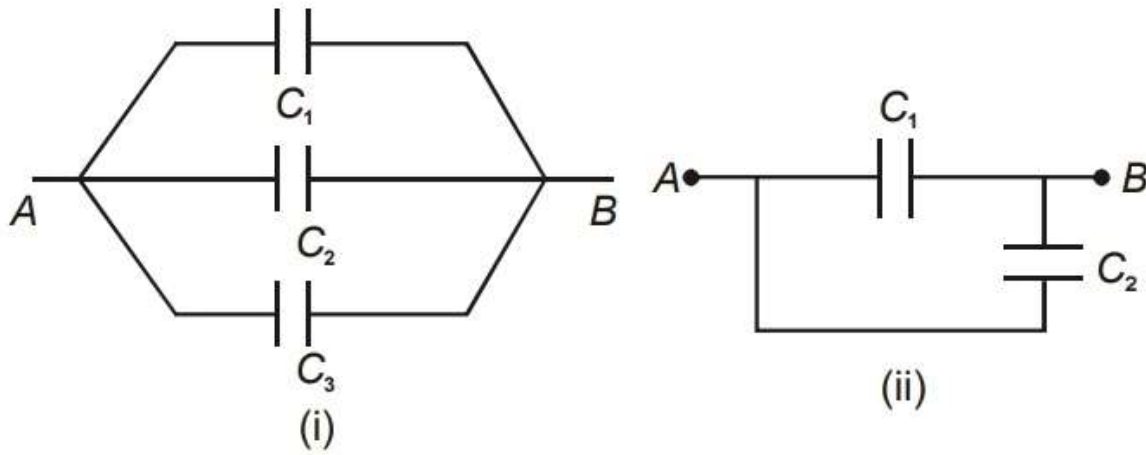
Series Grouping:

The arrangements shown in figure are examples of series grouping. When capacitors can be arranged in a row, so that there is no connection from in between two capacitors to any third capacitor, it is called a series combination. Or, when same charge flows through each capacitor connected.



$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

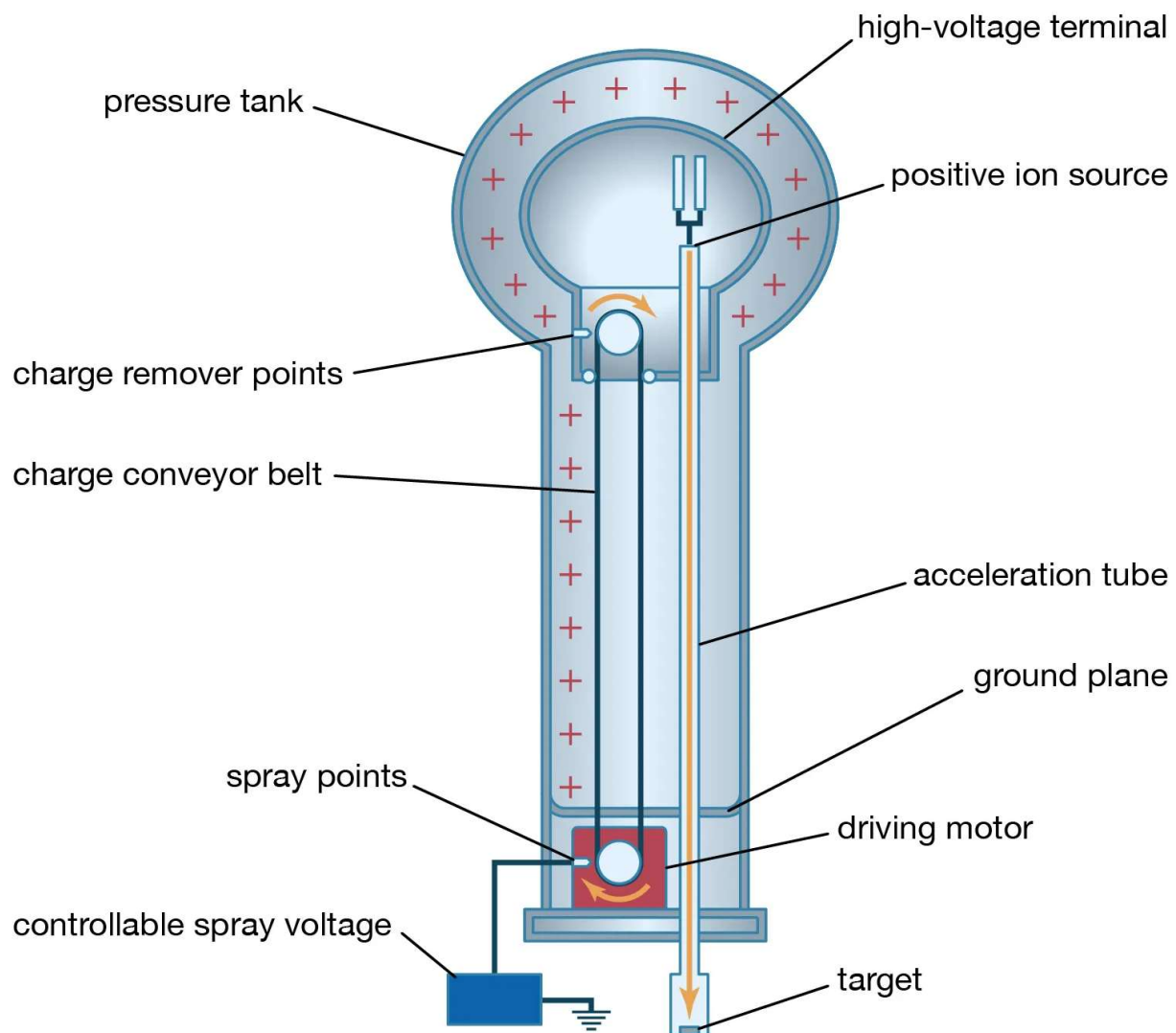
Parallel Grouping: The arrangements shown in figure are examples of parallel combination. When two or more capacitors are connected between two given points, they are said to be in parallel. Or, when capacitor bears same potential difference across it.



$$C = C_1 + C_2 + C_3$$

Van de Graaff Generator:

Van de Graaff generator is a machine that can built up voltages in order of a few million volts. The resultant electric fields are used to accelerate charged particles (proton, electrons, ions) to high energies required for experiments to examine small scale structure of matter.



Important Questions

Multiple Choice questions-

1. Which of the following statement is true?

- (a) Electrostatic force is a conservative force.
- (b) Potential at a point is the work done per unit charge in bringing a charge from any point to infinity.
- (c) Electrostatic force is non-conservative
- (d) Potential is the product of charge and work.

2. 1 volt is equivalent to

- (a) $\frac{\text{newton}}{\text{second}}$
- (b) $\frac{\text{newton}}{\text{coulomb}}$
- (c) $\frac{\text{joule}}{\text{coulomb}}$
- (d) $\frac{\text{joule}}{\text{second}}$

3. The work done in bringing a unit positive charge from infinite distance to a point at distance x from a positive charge Q is W . Then the potential at that point is

- (a) $\frac{WQ}{x}$
- (b) W
- (c) $\frac{W}{x}$
- (d) WQ

4. Consider a uniform electric field in the z -direction. The potential is a constant

- (a) for any x for a given z
- (b) for any y for a given z
- (c) on the x - y plane for a given z
- (d) all of these

5. Equipotential surfaces

- (a) are closer in regions of large electric fields compared to regions of lower electric fields.
- (b) will be more crowded near sharp edges of a conductor.
- (c) will always be equally spaced.
- (d) both (a) and (b) are correct.

6. In a region of constant potential

- (a) the electric field is uniform.
- (b) the electric field is zero.
- (c) there can be no charge inside the region.

(d) both (b) and (c) are correct.

7. A test charge is moved from lower potential point to a higher potential point. The potential energy of test charge will

(a) remain the same

(b) increase

(c) decrease

(d) become zero

8. An electric dipole of moment \vec{P} is placed in a uniform electric field \vec{E} . Then

(i) the torque on the dipole is $\vec{P} \times \vec{E}$

(ii) the potential energy of the system is $\vec{P} \cdot \vec{E}$

(iii) the resultant force on the dipole is zero. Choose the correct option.

(a) (i), (ii) and (iii) are correct

(b) (i) and (iii) are correct and (ii) is wrong

(c) only (i) is correct

(d) (i) and (ii) are correct and (iii) is wrong

9. If a conductor has a potential $V \neq 0$ and there are no charges anywhere else outside, then

(a) there must be charges on the surface or inside itself.

(b) there cannot be any charge in the body of the conductor.

(c) there must be charges only on the surface.

(d) both (a) and (b) are correct.

10. Which of the following statements is false for a perfect conductor?

(a) The surface of the conductor is an equipotential surface.

(b) The electric field just outside the surface of a conductor is perpendicular to the surface.

(c) The charge carried by a conductor is always uniformly distributed over the surface of the conductor.

(d) None of these.

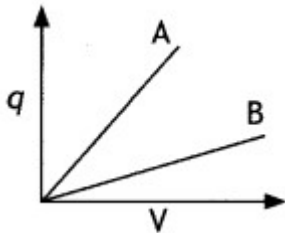
Very Short:

1. Express dielectric constant in terms of the capacitance of a capacitor.

2. On what factors does the capacitance of a parallel plate capacitor depend?

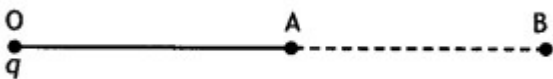
3. What is the ratio of electric field intensities at any two points between the plates of a capacitor?

4. Write a relation between electric displacement vector D and electric field E .
5. Write the relation between dielectric constant (K) and electric susceptibility χ_e .
6. A hollow metal sphere of radius 5 cm is charged such that the potential on its surface is 10 V. What is the potential at the center of the sphere? (CBSE AI 2011)
7. What is the geometrical shape of equipotential surfaces due to a single isolated charge? (CBSE Delhi 2013)
8. Draw the equipotential surfaces due to an isolated point charge. (CBSE Delhi 2019)
9. 'For any charge configuration, equipotential surface through a point is normal to the electric field'. Justify. (CBSE Delhi 2014)
10. The given graph shows the variation of charge ' q ' versus potential difference ' V ' for two capacitors C_1 and C_2 . Both the capacitors have the same plate separation but the plate area of C_2 is greater than that of C_1 . Which line (A or B) corresponds to C_1 and why? (CBSE AI 2014C)

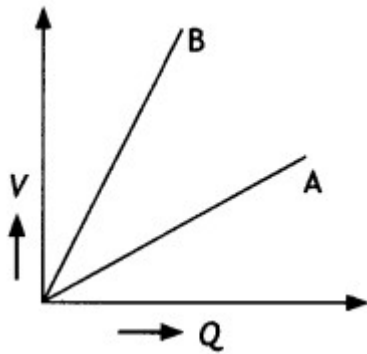


Short Questions:

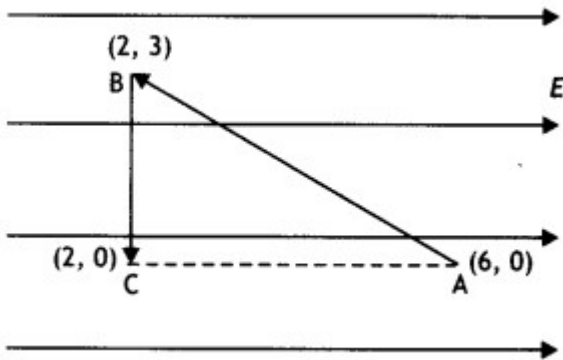
1. Draw a plot showing the variation of (i) electric field (E) and (ii) electric potential (V) with distance r due to a point charge Q . (CBSE Delhi 2012)
2. Two identical capacitors of 10 pF each are connected in turn (i) in series and (ii) in parallel across a 20 V battery. Calculate the potential difference across each capacitor in the first case and the charge acquired by each capacitor in the second case. (CBSE AI 2019)
3. A point charge ' q ' is placed at O as shown in the figure. Is $V_A - V_B$ positive, negative, or zero, if ' q ' is an (i) positive, (ii) negative charge? (CBSE Delhi 2011, 2016).



4. The graph shows the variation of voltage V across the plates of two capacitors A and B versus charge Q stored on them. Which of the two capacitors has higher capacitance? Give a reason for your answer.



5. A test charge 'q' is moved without acceleration from A to C along the path from A to B and then from B to C in electric field E as shown in the figure,



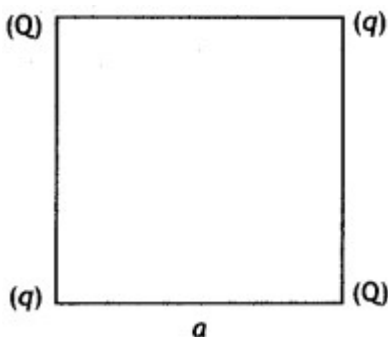
(i) Calculate the potential difference between A and C

(ii) At which point (of the two) is the electric potential more and why? (CBSE AI 2012)

6. A slab of material of dielectric constant K has the same area as that of the plates of a parallel plate capacitor but has the thickness $d/2$, where d is the separation between the plates. Find out the expression for its capacitance when the slab is inserted between the plates of the capacitor. (CBSE AI 2013)

7. Two-point charges q and $-2q$ are kept ' d ' distance apart. Find the location of the point relative to charge ' q ' at which potential due to this system of charges is zero. (CBSE AI 2014C)

8. Four-point charges Q , q , Q , and q are placed at the corners of a square of side ' a ' as shown in the figure.



Find the potential energy of this system. (CBSEAI, Delhi 2018)

Long Questions:

1. Two-point charges $2 \mu\text{C}$ and $-2 \mu\text{C}$ are placed at points A and B 6 cm apart.

(a) Draw the equipotential surfaces of the system.

(b) Why do the equipotential surfaces get closer to each other near the point charges?

(CBSEAI2O11C)

2.

(a) Obtain the expressions for the resultant capacitance when the three capacitors C_1 , C_2 , and C_3 are connected (i) in parallel and then (ii) in series.

(b) In the circuit shown in the figure, the charge on the capacitor of $4 \mu\text{F}$ is $16 \mu\text{C}$. Calculate the energy stored in the capacitor of $12 \mu\text{F}$ capacitance. (CBSE 2019C)

Assertion and Reason Questions-

1. For two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

- Both A and R are true, and R is the correct explanation of A.
- Both A and R are true, but R is not the correct explanation of A.
- A is true, but R is false.
- A is false, and R is also false.

Assertion (A): An electric field is preferred in comparison to magnetic field for detecting the electron beam in a television picture tube.

Reason (R): Electric field requires low voltage.

2. For two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

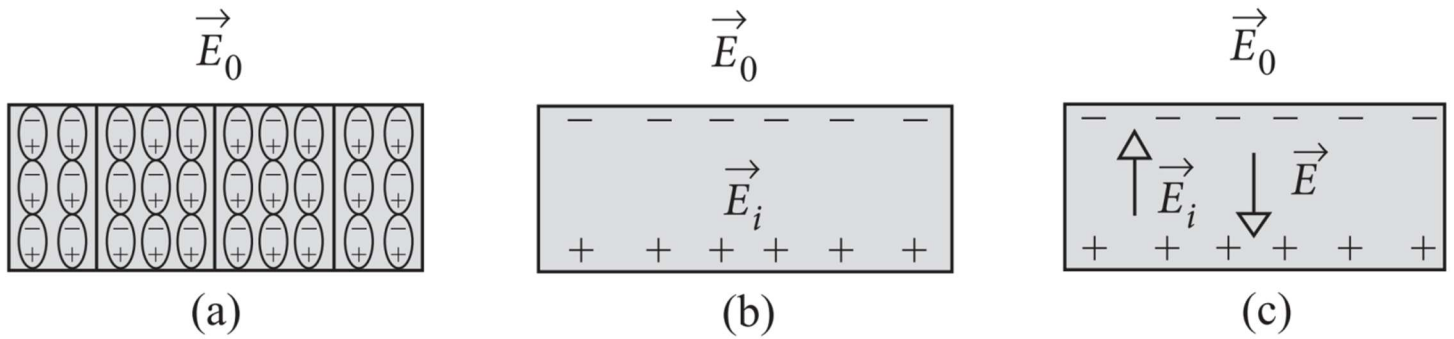
- Both A and R are true, and R is the correct explanation of A.
- Both A and R are true, but R is not the correct explanation of A.
- A is true, but R is false.
- A is false, and R is also false.

Assertion (A): An applied electric field will polarize the polar dielectric material.

Reason (R): In polar dielectrics, each molecule has a permanent dipole moment but these are randomly oriented in the absence of an externally applied electric field.

Case Study Questions-

1. When an insulator is placed in an external field, the dipoles become aligned. Induced surface charges on the insulator establish a polarization field \vec{E}_i in its interior. The net field \vec{E} in the insulator is the vector sum of \vec{E}_0 and \vec{E}_i as shown in the figure.



On the application of external electric field, the effect of aligning the electric dipoles in the insulator is called polarisation, and the field is known as the polarisation field.

The dipole moment per unit volume of the dielectric is known as polarisation \vec{P} . For linear isotropic dielectrics, $\vec{P} = \chi \vec{E}$, where χ = electrical susceptibility of the dielectric medium.

- (i) Which among the following is an example of polar molecule?
- O₂
 - H₂
 - N₂
 - HCl
- (ii) When air is replaced by a dielectric medium of constant K, the maximum force of attraction between two charges separated by a distance:
- Increases K times.
 - Remains unchanged.
 - Decreases K times.
 - Increases 2K times.
- (iii) Which of the following is a dielectric?
- Copper.
 - Glass.
 - Antimony (Sb).
 - None of these.
- (iv) For a polar molecule, which of the following statements is true?
- The centre of gravity of electrons and protons coincide.
 - The centre of gravity of electrons and protons do not coincide.
 - The charge distribution is always symmetrical.
 - The dipole moment is always zero.

- (v) When a comb rubbed with dry hair attracts pieces of paper. This is because the?
- Comb polarizes the piece of paper.
 - Comb induces a net dipole moment opposite to the direction of field.
 - Electric field due to the comb is uniform.
 - Comb induces a net dipole moment perpendicular to the direction of field.

2. This energy possessed by a system of charges by virtue of their positions. When two like charges lie infinite distance apart, their potential energy is zero because no work has to be done in moving one charge at infinite distance from the other.

In carrying a charge q from point A to point B, work done $W = q(V_A - V_B)$. This work may appear as change in $\frac{KE}{PE}$ of the charge. The potential energy of two charges q_1 and q_2 at a distance r in air is $\frac{q_1 q_2}{4\pi\epsilon_0 r}$.

It is measured in joule. It may be positive, negative or zero depending on the signs of q_1 and q_2 .

i. Calculate work done in separating two electrons from a distance of

1m to 2m in air, where e is electric charge and k is electrostatic force constant.

- ke^2
 - $\frac{e^2}{2}$
 - $-\frac{ke^2}{2}$
 - Zero
- ii. Four equal charges q each are placed at four corners of a square of side a each. Work done in carrying a charge $-q$ from its centre to infinity is:

- Zero
- $\frac{\sqrt{2}q^2}{\pi\epsilon_0 a}$
- $\frac{\sqrt{2}q}{\pi\epsilon_0 a}$
- $\frac{q^2}{\pi\epsilon_0 a}$

iii. Two points A and B are located in diametrically opposite directions of a point charge of $+2\mu\text{C}$ at distances 2m and 1m respectively from it. The potential difference between

A and B is:

- a. $3 \times 10^3\text{V}$
- b. $6 \times 10^4\text{V}$
- c. $-9 \times 10^3\text{V}$
- d. $-3 \times 10^3\text{V}$

iv. Two point charges $A = + 3\text{nC}$ and $B = + 1\text{nC}$ are placed 5cm apart in air.

The work done to move charge B towards A by 1cm is:

- a. $2.0 \times 10^{-7}\text{J}$
- b. $1.35 \times 10^{-7}\text{J}$
- c. $2.7 \times 10^{-7}\text{J}$
- d. $12.1 \times 10^{-7}\text{J}$

v. A charge Q is placed at the origin. The electric potential due to this charge at a given point in space is V . The work done by an external force in bringing another charge q from infinity up to the point is:

- a. $\frac{V}{q}$
- b. Vq
- c. $V + q$
- d. V

✓ Answer Key:

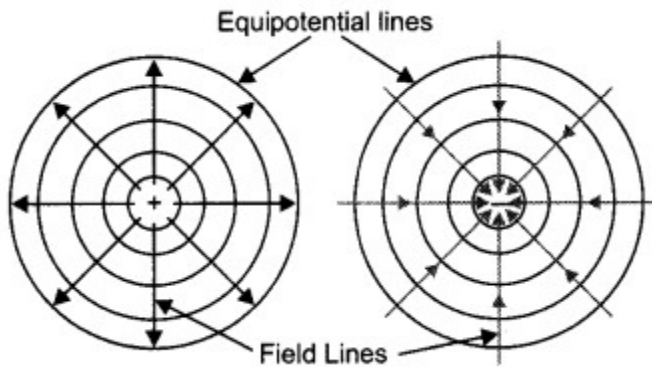
Multiple Choice Answers-

1. Answer: a
2. Answer: c
3. Answer: b
4. Answer: d
5. Answer: d
6. Answer: d
7. Answer: c
8. Answer: b
9. Answer: c
10. Answer: d

Very Short Answers:

1. Answer: It is given by the expression $K = \frac{C}{C_0}$ where C is the capacitance of the capacitor with dielectric and C_0 is the capacitance without the dielectric.
2. Answer:
 - Area of plates,
 - The separation between the plates and
 - Nature of dielectric medium between the plates.
3. Answer: The ratio is one, as the electric field is the same at all points between the plates of a capacitor.
4. Answer:

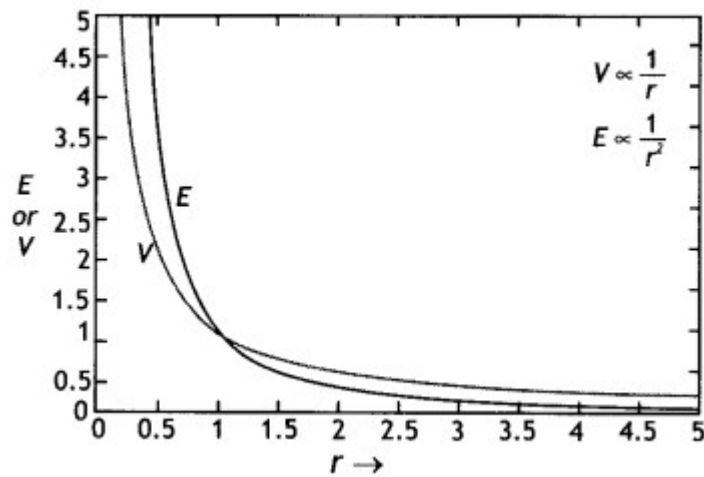
$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$
5. Answer: $K = 1 + \chi_e$
6. Answer: 10 V
7. Answer: Concentric circles.
8. Answer: These areas are shown.



9. Answer: This is because work done in moving a charge on an equipotential surface is zero. This is possible only if the equipotential surface is perpendicular to the electric field.
10. Answer: Since $C = \epsilon_0 A/d$, since the area for C_2 is more, therefore capacitance of C_2 is more. From the graph greater the slope greater is than the capacitance, therefore, graph A belongs to capacitor C_2 . While graph B belongs to capacitance C_v .

Short Questions Answers:

Answer: The plot is as shown.



Answer:

(i) Since the two capacitors have the same capacitance, therefore, the potential will be divided amongst them. Hence $V = 10$ V each

(ii) Since the capacitors are connected in parallel, therefore, potential difference = 20 V

Hence charge $Q = CV = 10 \times 20 = 200$ pC

Answer:

$$\text{If } V_A - V_B = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{OA} - \frac{1}{OB} \right)$$

As $OA < OB$

\therefore If q is positive then $V_A - V_B$ is positive and

if q is negative $V_A - V_B$ is also negative.

Answer:

Capacitor A has higher capacitance. We know that capacitance $C = Q/V$.

For capacitor A

$$C_A = \frac{Q}{V_A}$$

For capacitor B

$$C_B = \frac{Q}{V_B}$$

As $V_B > V_A$

$\therefore C_B < C_A$

Thus, capacitance of A is higher.

Answer:

(i) $dV = -E dr = -E(6 - 2) = -4E$

(ii) Electric potential is more at point C as $dV = -E dr$, i.e. the electric potential decreases in the direction of the electric field.

Answer:

Given $t = d/2$, $C = ?$

We know that when a dielectric of thickness 't' is inserted between the plates of a capacitor, its capacitance is given by

$$C = \frac{\epsilon_0 A}{d - t + \frac{t}{K}}$$

Hence we have

$$C = \frac{\epsilon_0 A}{d - \frac{d}{2} + \frac{d}{2K}} = \frac{2K\epsilon_0 A}{d(1+K)}$$

Answer:

Let the potential be zero at point P at a distance x from charge q as shown



Now potential at point P is

$$V = \frac{kq}{x} + \frac{k(-2q)}{d+x} = 0$$

Solving for x we have

$$x = d$$

Answer:

The potential energy of the system

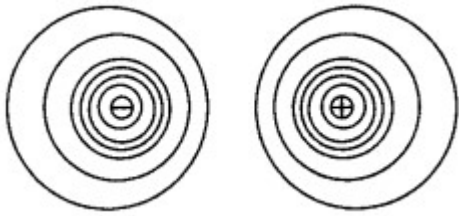
$$U = \frac{1}{4\pi\epsilon_0} \left(4 \frac{qQ}{a} + \frac{q^2}{a\sqrt{2}} + \frac{Q^2}{a\sqrt{2}} \right)$$

$$U = \frac{1}{4\pi\epsilon_0 a} \left(4qQ + \frac{q^2}{\sqrt{2}} + \frac{Q^2}{\sqrt{2}} \right)$$

Long Questions Answers:

1. Answer:

(a) The diagram is as shown.



(b) We know that $E = -dV/dr$

Therefore, $dr = -dV/E$

Since near the charge, electric field E is large, dr will be less.

2. Answer:

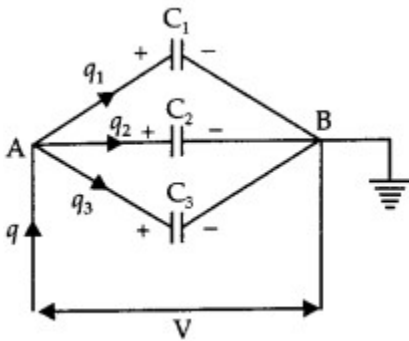
(i) Parallel combination of three capacitors.

Let three capacitors of capacitances C_1 , C_2 , and C_3 be connected in parallel, and potential difference V be applied across A and B. If q be total charge flowing in the circuit and q_1 , q_2 and q_3 be charges flowing across.

C_1 , C_2 , and C_3 respectively, then

$$q = q_1 + q_2 + q_3$$

$$\text{or } q = C_1V + C_2V + C_3V \dots (i)$$



If C_P is the capacitance of the arrangement in parallel, then

$$q = C_P V$$

So equation (i) becomes

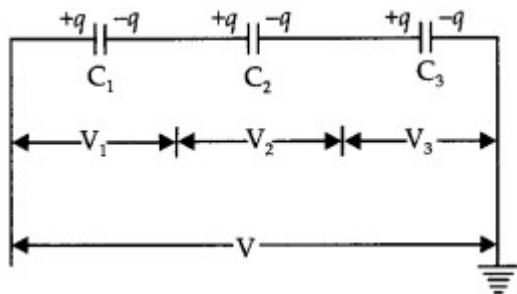
$$C_P V = C_1 V + C_2 V + C_3 V$$

Or

$$C_P = C_1 + C_2 + C_3$$

(ii) Series combination of three capacitors Let three capacitors C_1 , C_2 , and C_3 be connected in series. Let q charge be flowing through the circuit.

If V_1 , V_2 , and V_3 be potential differences across the plates of the capacitor and V be the potential difference across the series combination, then



$$V = V_1 + V_2 + V_3$$

Or

$$V = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} \dots (i)$$

If C_s is the capacitance of series combination, then $V = \frac{q}{C_s}$.

So the equation (i) becomes

$$\frac{q}{C_s} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

Or

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Charge q across $4 \mu\text{F}$ Capacitor is $16 \mu\text{C}$ Potential difference across the capacitor of capacitance $4 \mu\text{F}$ will be

$$V = \frac{q}{C} = \frac{16 \mu\text{C}}{4 \mu\text{F}} = \frac{16 \times 10^{-6} \text{C}}{4 \times 10^{-6} \text{F}} = 4\text{V}$$

\therefore Potential across $12 \mu\text{F}$ Capacitors

$$= 12\text{V} - 4\text{V} = 8\text{V}$$

Energy stored in the capacitors of capacitance $C = 12 \mu\text{F}$

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \times 12 \times 10^{-6} \times 8^2 \text{ joule}$$

$$= 384 \times 10^{-6} \text{ J} = 384 \mu\text{J}$$

Assertion and Reason Answers-

1. (d) A is false, and R is also false.

Explanation:

If electric field is used for detecting the electron beam, then very high voltage will have to be applied and very long tube will have to be taken.

2. (b) Both A and R are true, but R is not the correct explanation of A.

Explanation:

If a material contain polar molecules, they will generally be in random orientations when no electric field is applied. An applied electric field will polarize the material by orienting the dipole moment of polar molecules.

Case Study Answers-

1. Answer :

(i) (d) HCl

Explanation:

In polar molecule the centres of positive and negative charges are separated even when there is no external field. Such molecule have a permanent dipole moment. Ionic molecule like HCl is an example of polar molecule.

(ii) (c) Decreases K times.

Explanation:

$$\text{As } F_m = \frac{F_0}{K}$$

∴ The maximum force decreases by Klimes.

(iii) (b) Glass.

(iv) (b) The centre of gravity of electrons and protons do not coincide.

Explanation:

A polar molecule is one in which the centre of gravity for positive and negative charges are separated.

(v) (a) Comb polarizes the piece of paper.

2. Answer :

i. (c) $-\frac{ke^2}{2}$

Explanation:

$$W = (\text{P.E.})_{\text{final}} - (\text{P.E.})_{\text{initial}}$$

$$= \frac{ke^2}{2} - \frac{ke^2}{1} = \frac{-ke^2}{2}$$

ii. (b) $\frac{\sqrt{2}q^2}{\pi\epsilon_0 a}$

Explanation:

Potential at the centre of the square due to four equal charges q at four corners,

$$V = \frac{4q}{\frac{4\pi\epsilon_0(a\sqrt{2})}{2}} = \frac{\sqrt{2}q}{\pi\epsilon_0 a}$$

$$\begin{aligned} W_{0 \rightarrow \infty} &= -W_{0 \rightarrow \infty} = -(-q)V \\ &= \frac{\sqrt{2}q^2}{\pi\epsilon_0 a} \end{aligned}$$

iii. (c) $-9 \times 10^3 \text{V}$

Explanation:

Here, $q = 2\mu\text{C} = 2 \times 10^{-6} \text{C}$, $r_A = 2\text{m}$, $r_B = 1\text{m}$.

$$\begin{aligned} \therefore V_A - V_B &= \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r_A} - \frac{1}{r_B} \right] \\ &= 2 \times 10^{-6} \times 9 \times 10^9 \left[\frac{1}{2} - \frac{1}{1} \right] \end{aligned}$$

$$V = -9 \times 10^3 \text{V}.$$

iv. (b) $1.35 \times 10^{-7} \text{J}$

Explanation:

Required work done = Change in potential energy of the system,

$$\begin{aligned} W &= U_f - U_i = k \frac{q_1 q_2}{r_f} - k \frac{q_1 q_2}{r_i} \\ &= k q_1 q_2 \left[\frac{1}{r_f} - \frac{1}{r_i} \right] \\ \therefore W &= (9 \times 10^9)(3 \times 10^{-9} \times 1 \times 10^{-9}) \\ &\times \left[\frac{1}{4 \times 10^{-2}} - \frac{1}{5 \times 10^{-2}} \right] \\ &= 27 \times 10^{-7} \times (0.05) = 1.35 \times 10^{-7} \text{J}. \end{aligned}$$

v. (b) Vq