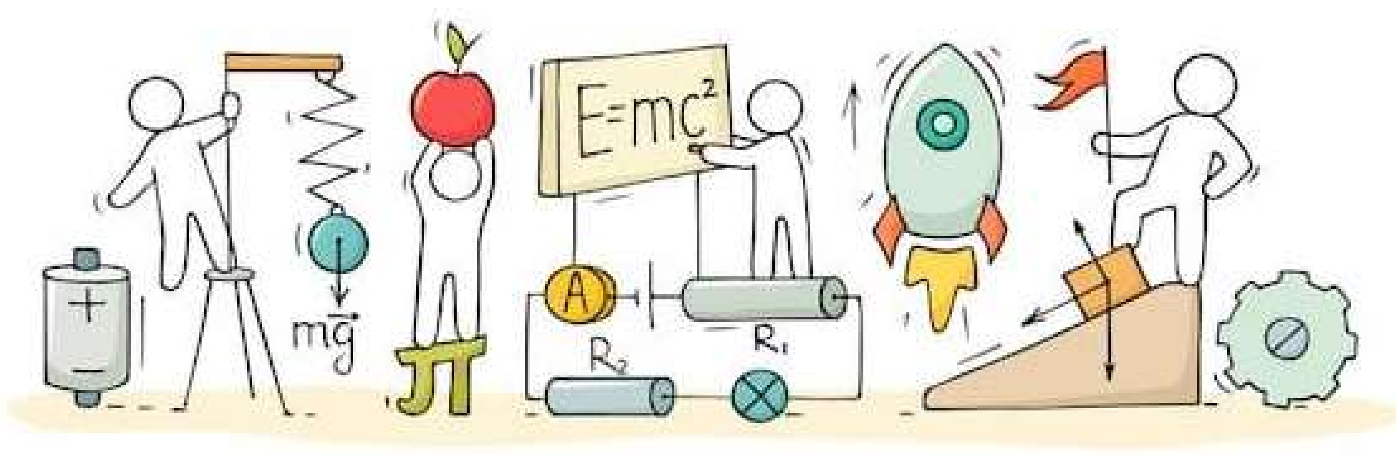
PHYSICS

Chapter 2: Units and Measurements



Units and Measurements

Top Formulae

Mean value	$a_{mean} = (a_1 + a_2 + a_3 + \ldots + a_n)/n$ or $a_{mean} = \sum_{i=1}^{n} a_i / n$
Errors in the individual measurement values from the true value (Absolute Error)	$\Delta a_1 = a_1 - a_{mean}$ $\Delta a_2 = a_2 - a_{mean}$. = . . . = . . $\Delta a_n = a_n - a_{mean}$
Mean absolute error	$\Delta a_{mean} =$ $(\lvert\Delta a_1\rvert + \lvert\Delta a_2\rvert + \lvert\Delta a_3\rvert + \ldots + \lvert\Delta a_n\rvert)/n$ $= \sum_{i=1}^{n} \lvert a_i \rvert / n$
Relative error	$\Delta a_{mean}/a_{mean}$
Percentage error	$\delta a = (\Delta a_{mean}/a_{mean}) \times 100\%$
Error of a sum or a difference	$\pm \Delta Z = \pm \Delta A \pm \Delta B$ or The maximum value of the error ΔZ is $\Delta A + \Delta B$.
Error of a product or a quotient	$\Delta Z/Z = (\Delta A/A) + (\Delta B/B)$
Error in case of a measured quantity raised to a power	If $Z = A^p B^q / C^r$, then $\Delta Z/Z = p(\Delta A/A) + q(\Delta B/B) + r(\Delta C/C)$

Top Concepts

- The system of units used around the world is the International System of Units or SI.
- The units for the base quantities are called fundamental or base units. The units of all other physical quantities can be expressed as a combination of base units. Such units obtained are called derived units.

Base quantity	Name	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Amount of substance	mole	Mol
Luminous intensity	candela	cd

- Other physical quantities, derived from the base quantities, can be expressed as a combination of the base units and are called derived units. A complete set of units, both fundamental and derived, is called a system of units.
- In computing any physical quantity, the units of derived quantities involved in the relationship(s) are treated as algebraic quantities till the desired units are obtained.
- The apparent shift in the position of the object against the reference point in the background is called parallax.
- Parallax is caused whenever there is a change in the point of observation. The distance between the two points of observation is called the basis. Let the basis be b and the angle subtended by it at some point be θ, then the distance of the point $D = \frac{b}{\theta}$.
- The size of the molecules of a solution = volume of film/area of film
- The unit used to measure the size of the nucleus of an atom is fermi which is 10^{-15} m.
- The unit used to measure the distance between the Earth and the Sun is the astronomical unit (AU).
- The smallest value measured by an instrument is called its least count. The least count of the vernier callipers is 0.01 cm and that of the screw gauge is 0.001 cm.
- The different types of errors are absolute error, relative error and percentage error.

- True value is the mean of all the observed readings.
- The absolute error is the magnitude of the difference between the individual measured value and the true value.

  Absolute error = Measured value − True value

- The fractional error is the ratio of the mean absolute error to the true value. It is also known as relative error.

$$\text{Relative error} = \frac{\text{Mean absolute error}}{\text{True value}}$$

- Direct and indirect methods can be used for the measurement of physical quantities. In measured quantities, while expressing the result, the accuracy and precision of measuring instruments along with errors in measurements should be taken into account.
- Significant figures in measured or observed values are the number of reliable digits plus the first uncertain digit.
- Rules to identify significant figures:

  i. All non-zero digits are significant. Powers of ten are not counted in significant figures. Example: 1.7×10^5 has two significant figures.

  ii. In a number with a decimal, the zeroes appearing to the left of a digit are not counted in significant figures. Example: 0.002 has only one significant figure in it.

  iii. In a number with a decimal, the number of zeroes at the end is counted in significant figures. Example: 1.700 has four significant figures.

  iv. Shifting the position of the decimal does not change the number of significant figures. Example: 2.340 and 234.0 have four significant figures.

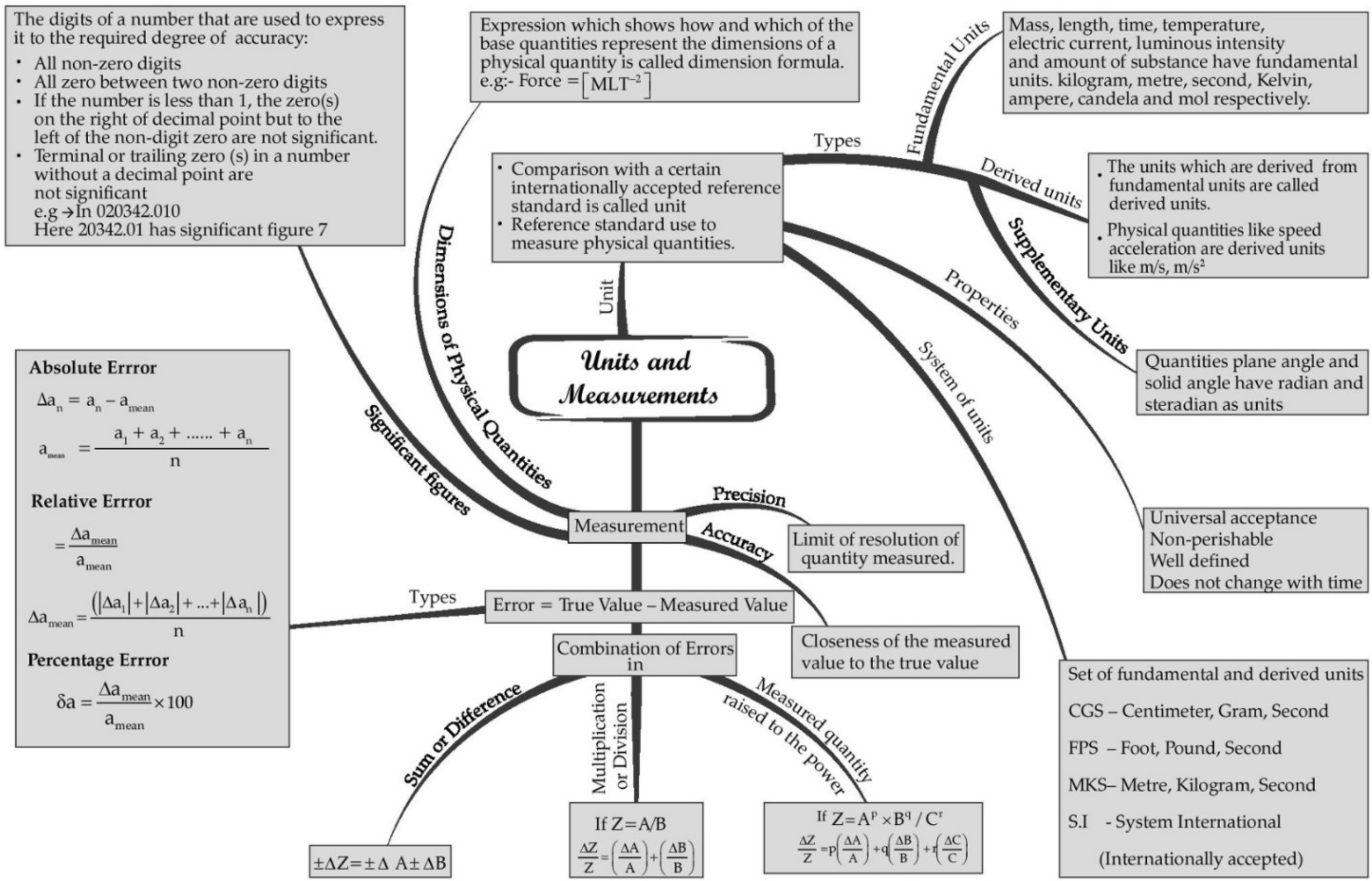
  v. All the zeros between two non-zero digits are significant, no matter where the decimal place if at all. Examples: 203.4 cm has four significant digits, 2.05 has three significant digits.

  vi. The terminal or trailing zeros in a number without a decimal point are not significant. Thus, 125 m = 12500 cm = 125000 mm has three significant figures.

- Changing the units does not change the number of significant figures.

- Dimensions of a physical quantity are the powers (or exponents) to which the base quantities are raised to represent that quantity.
- **Dimensional formula:** The expression which shows how and which of the base quantities represent the dimensions of a physical quantity.
- Applications of dimensional analysis:
  i. To derive a physical equation.
  ii. To verify if the given equation is dimensionally correct.
  iii. To find the dimensions of an unknown parameter in the equation.

CHAPTER - 2 : UNITS AND MEASUREMENTS



Important Questions

Multiple Choice questions-

1. Electron volt is a unit of

(a) charge

(b) potential difference

(c) energy

(d) magnetic force

2. Light year is a unit of

(a) time

(b) distance

(c) sunlight intensity

(d) mass

3. Which of the following pairs has the same dimensions?

(a) specific heat and latent heat

(b) impulse and momentum

(c) surface tension and force

(d) moment of inertia and torque

4. Which of the following sets of quantities has the same dimensional formula?

(a) Frequency, angular frequency and angular momentum

(b) Surface tension, stress and spring constant

(c) Acceleration, momentum and retardation

(d) Work, energy and torque

5. If C and R denote capacitance and resistance respectively, what will be the dimensions of C x R?

(a) $[M^0L^0TA^0]$

(b) $[ML^0TA^{-2}]$

(c) $[ML^0TA^2]$

(d) $[MLTA^{-2}]$

6. A particle starting from the origin (0, 0) moves in a straight line in the (x, y) plane. Its coordinates at a later time are (The path of the particle makes with the x-axis an angle of

(a) 300

(b) 450

(c) 600

(d) 0

7. Resolution is

(a) a measure of the bias in the instrument

(b) None of these

(c) the smallest amount of input signal change that the instrument can detect reliably

(d) a measure of the systematic errors

8. Fundamental or base quantities are arbitrary. In SI system these are

(a) length, force, time, electric current, thermodynamic temperature, amount of substance, and luminous intensity

(b) length, mass, time, electric current, thermodynamic temperature, amount of substance, and luminous intensity

(c) as length, mass, time, electric charge, thermodynamic temperature, amount of substance, and luminous intensity

(d) length, mass, force, electric current, thermodynamic temperature, amount of substance, and luminous intensity

9. Unit for a fundamental physical quantity is

(a) defined as best of various reference standards

(b) the smallest measurable value of the physical quantity

(c) defined as average various reference standards

(d) reference standard for the physical quantity

10. The volume of a cube in m^3 is equal to the surface area of the cube in m^2. The volume of the cube is

(a) 64 m^3

(b) 216 m^3

(c) 512 m^3

(d) 196 m^3

Very Short:

1. If the size of the atom were enlarged to the tip of the sharp pin, how large would the height of Mount Everest be?
2. What does the LASER mean?

3. If the Universe were shrunk to the size of the Earth, how large would the Earth be on this scale?
4. A research worker takes 100 careful readings in an experiment. If he repeats the same experiment by taking 400 readings, then by what factor will the probable error be reduced?
5. What is the number of significant figures in 0.06070?
6. The density of a cube is calculated by measuring the length of one side and its mass. If the maximum errors in the measurement of mass and length are 3% and 2% respectively, then what is the maximum possible error in the measurement of density?
7. The mass of a body as measured by two students is given as 1.2 kg and 1.23 kg. Which of the two is more accurate and why?
8. Do the inertial and gravitational masses of ordinary objects differ in magnitude?
9. Are S.I. units Coherent? Why?
10. Do A.U. And Å represents the same magnitudes of distance?

Short Questions:

1. If the size of a nucleus is scaled up to the tip of a sharp pin, what roughly is the size of an atom?
2. (a) What do you mean by physical quantity?

   (b) What do you understand by:

   (i) Fundamental physical quantities?

   (ii) Derived physical quantities?
3. (a) Define the unit of a physical quantity.

   (b) Define

   (i) Fundamental units.

   (ii) Derived units.
4. Define one Candela.
5. What is the advantage of choosing wavelength of light radiation as standard of length?
6. Which type of phenomenon can be used as a measure of time? Give two examples of it.
7. Find the number of times the heart of a human being beats in 10 years. Assume that the heartbeats once in 0.8s.
8. Why it is not possible to establish a physical relation involving more than three variables using the method of dimensions?

Long Questions:

1. State the rules for writing the units of physical quantities in the S.I. system.
2. Explain the Triangular method.
3. What are the uses of dimensional analysis? Explain each of them.

Assertion Reason Questions:

1. **Directions:** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

   (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
   (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
   (c) Assertion is correct, reason is incorrect
   (d) Assertion is incorrect, reason is correct

   Assertion: Dimensional constants are the quantities whose values are constant.
   Reason: Dimensional constants are dimensionless.

2. **Directions:** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

   (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
   (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
   (c) Assertion is correct, reason is incorrect
   (d) Assertion is incorrect, reason is correct

   Assertion: Parallax method cannot be used for measuring distances of stars more than 100 light years away.

   Reason: Because parallax angle reduces so much that it cannot be measured accurately.

Case Study Questions:

1. Measurement of Physical Quantity All engineering phenomena deal with definite and measured quantities and so depend on the making of the measurement. We must be clear and precise in making these measurements. To make a measurement, magnitude of the physical quantity (unknown) is compared. The record of a measurement consists of three parts, i.e., the dimension of the quantity, the unit which represents a standard quantity and a number which is the ratio of the measured quantity to the standard quantity.

   1. A device which is used for measurement of length to an accuracy of about 10-^{5m}, is

      (a) Screw gauge

      (b) Spherometer

      (c) Vernier callipers

      (d) Either (a) or (b)

2. Which of the technique is not used for measuring time intervals?
   (a) Electrical oscillator
   (b) Atomic clock
   (c) Spring oscillator
   (d) Decay of elementary particles
3. The mean length of an object is 5cm. Which of the following measurements is most accurate?
   (a) 4.9cm
   (b) 4.805cm
   (c) 5.25cm
   (d) 5.4cm
4. If the length of rectangle l = 105.cm, breadth b = 21.cm and minimum possible measurement by scale = 01.cm, then the area is
   (a) 22.0cm^2
   (b) 21.0cm^2
   (c) 22.5cm^2
   (d) 21.5cm^2
5. Age of the universe is about 1010 yr., whereas the mankind has existed for 106 yr. For how many seconds would the man have existed if age of universe were 1 day?
   (a) 9.2 s
   (b) 10.2 s
   (c) 8.6 s
   (d) 10.5 s

2. Normally, the reported result of measurement is a number that includes all digits in the number that are known reliably plus first digit that is uncertain. The digits that are known reliably plus the first uncertain digit are known as significant digits or significant figures.

   e.g., When a measured distance is reported to be 374.5m, it has four significant figures 3, 7, 4 and 5. The figures 3, 7, 4 are certain and reliable, while the digit 5 is uncertain. Clearly, the digits beyond the significant digits reported in any result are superfluous.

   i. In 4700m, significant digits are
      (a) 2

(b) 3

(c) 4

(d) 5

ii. To determine the number of significant figures, scientific notation is

(a) a^b

(b) ab $\times 10^b$

(c) a $\times 10^2$

(d) a $\times 10^4$

iii. 5.74 g of a substance occupies $1.2cm^3$ Express its density by keeping the significant figures in view.

(a) 4.9 g cm^{-3}

(b) 5.2 g cm^{-3}

(c) 4.8 g cm^{-3}

(d) 4.4 g cm^{-3}

iv. Choose the correct option.

(a) Change in unit does not change the significant figure.

(b) 4 700. m=4700 mm, here there is a change of significant number from 4 to 2 due to change in unit.

(c) 4700 4 700 103 m =´ . m, here there is change in numbers of significant numbers.

(d) Change in unit changes the number of significant figure.

v. Consider the following rules of significant figures.

I. All the non-zero digits are significant.

II. All the zeroes between two non-zero digits are significant.

III. The terminal or trailing zero(s) in a number without a decimal point are significant.

Which of the above statement(s) is/are? correct?

(a) I and II

(b) II and III

(c) I and III

(d) All of these

✓ Answer Key:

Multiple Choice Answers-

1. Answer: (c) energy
2. Answer: (b) distance
3. Answer: (b) impulse and momentum
4. Answer: (d) Work, energy, and torque
5. Answer: (a) $[M^0L^0TA^0]$
6. Answer: (c) 600
7. Answer: (d) a measure of the systematic errors
8. Answer: (b) length, mass, time, electric current, thermodynamic temperature, amount of substance, and luminous intensity
9. Answer: (d) reference standard for the physical quantity
10. Answer: (b) 216 m^3

Very Short Answers:

1. Answer: 10^{10} m.
2. Answer: It stands for Light Amplification by Stimulated Emission of Radiation.
3. Answer: 10^{-11} m (size of an atom.).
4. Answer: By a factor of 4.
5. Answer: 4.
6. Answer: 3% + 3 × 2% = 9%.
7. Answer: The second measurement is more accurate as it has been made to the second decimal point.
8. Answer: No.
9. Answer: Yes, because all the derived units in this system can be obtained by multiplying or dividing a certain set of basic units.
10. Answer: No, 1 A.U. = 1.496 × 10^{11} m and 1 Å = 10^{10} m.

Short Questions Answers:

1. Answer: The size of a nucleus is in the range of 10^{-15} m to 10^{-14} m. The tip of a sharp pain may be taken to be in the range of 10^{-5} m to 10^{-4} m. Thus, we are scaling up the size of the nucleus by a factor of $10^{-5}/10^{-15} = 10^{10}$. An atom roughly of size 10-10 m will be scaled up to a rough size of $10^{-10} \times 10^{10}$ = 1 m. Thus, nucleus in an atom is as small in size as the tip of a sharp pin placed at the center of a sphere of radius about a meter.
2. Answer: It is defined as a quantity that can be measured, e.g., mass, length, time, etc.

(b)

(i) They are defined as those quantities which cannot be expressed in terms of other quantities and are independent of each other, e.g., mass, length, time.

(ii) They are defined as the quantities which can be expressed in terms of fundamental quantities, e.g., velocity, acceleration, density, pressure, etc.

3. Answer: It is defined as the reference standard used to measure a physical quantity.

(b)

(i) They are defined as the units of fundamental quantities. They are independent of each other and are expressed by writing the letter of the fundamental quantity in a parenthesis. e.g., Fundamental units of mass, length and time are [M], [L], [T] respectively.

(ii) They are defined as those units which can be derived from fundamental units. They are expressed by writing the symbol of a derived quantity in a parenthesis.

e.g., D.U. of velocity = [u]

acceleration = [a]

pressure = [P]

work = [W] and so on.

4. Answer: It is defined as the luminous intensity in a perpendicular direction of a surface of $\frac{1}{600,000}$ square meter area of a black body at a temperature of freezing platinum (1773°C) under a pressure of 101,325 N/m^2.

5. Answer:

- It can be easily made available in any standard laboratory as Krypton is available everywhere.
- It is well defined and does not change with temperature, time, place or pressure, etc.
- It is invariable.
- It increases the accuracy of the measurement of length (1 part in 10^9).

6. Answer: Any phenomenon that repeats itself regularly at equal intervals of time can be used to measure time.

The examples are:

- Rotation of earth – the time interval for one complete rotation is called a day.
- Oscillations of a pendulum.

7. Answer: In 0.8 s, the human heart makes one beat.

∴ In 1 s, the human heart makes = $\frac{1}{0.8} = \frac{10}{8}$ beats.

∴ In 10 years, the human heart makes

= $\frac{10}{8}$ × 365 × 24 × 60 × 60 beats.

= 3.942×10^8 beats.

8. Answer: The dimensional analysis fails to derive a relation involving more than three unknown variables. The reason is that there will be more than three unknown factors in that case whose values cannot be determined from the three relations which we get by comparing the powers of M, L, and T.

Long Questions Answers:

1. Answer: While writing the units of physical quantities following rules are followed with S.L units:

   (1) The S.l. units are written in the form of symbols after the number i.e., number of time, the unit is contained in the physical quantity so that physical quantity = nu

   With symbols, certain rules are laid down:

   - Units in symbols are never written in plural i.e., meters is only m and not ms, years is y.
   - The units based on the name of the scientists are written beginning with small letters and with capital letters in symbolic form viz, weber (Wb), newton (N), etc.
   - No full stop is used at the end of the symbol.
   - Symbols of units not based on the name of scientists are written as small letters viz. kilogram (kg), second (s), etc.

   (2) Bigger and smaller number of units are represented with symbols corresponding to the power of 10 viz. 106 is mega (M), 10^{12} is Tera (T), 10^{-3} is milli (m), 10^{-9} is nano (n), etc.

   (3) All units are written in numerator viz. kg/m^3 is kg m, Nm^2c^2.

   (4) The units are written within parenthesis in graphs below the corresponding taxes viz. (ms^{-1}) and (s) in the velocity-time graph.

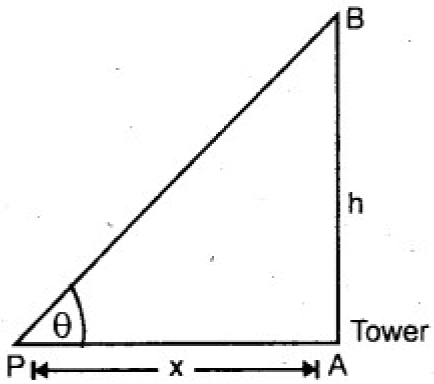
   (5) Units of a similar physical quantity can be added or subtracted.

2. Answer: It is used to measure the distance of an accessible or inaccessible hill or a tower by measuring the angle which the object makes at point P (say)

Let x = distance y of point P from the foot of tower = PA .

∴ h = x tan θ

It is also used to measure the distance of an inaccessible object eg. a tree on the other bank of a river.



Let h = height of the inaccessible object.

Let θ1, θ2 = be the angle made at P and Q by the object.

Let PA = d, PQ = x.

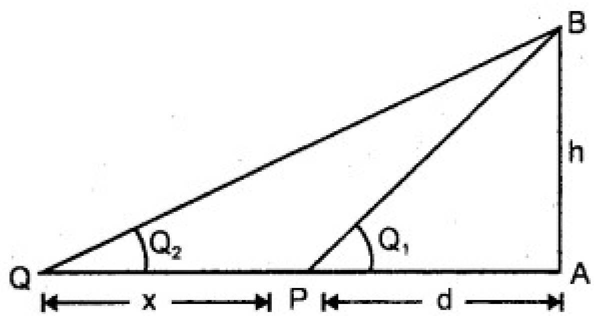
∴ In ΔPAB and ΔQAB,

$$d = h \cot \theta_1 \quad \dots (i)$$

and $$d + x = h \cot\theta_2 \quad \dots (ii)$$

(ii) and (i) gives, $$x = h(\cot\theta_2 - \cot\theta_1)$$

∴ $$h = x/(\cot\theta_2 - \cot\theta_1).$$



3. Answer:

   Dimensional analysis is used for:

   (a) checking the dimensional correctness of the given physical equation or relation.

   (b) converting one system of units to another system.

   (c) deriving the relationship between various physical quantities.

   (a) checking of the dimensional correctness of a physical relationship is done by using the principle of homogeneity of dimensions. If the dimensions of M, L, T of each term on R.H.S. are equal to the dimensions of M, L, T of each term on L.H.S., then the given-physical relation is dimensionally correct, otherwise wrong.

   (b) conversion: It is based on the fact that the magnitude of a physical quantity remains the same whatever may be the system of units, i.e., $n_1u_1 = n_2u_2$.

or $n_2 = n_1 \frac{u_1}{u_2}$

where $u_1 = M^a_1 L^b_1 T^c_1$

and $u_2 = M^a_2 L^b_2 T^c_2$

are the units of M, L, T in the first and second system of units of a physical quantity having dimensions of M, L, T, and a, b, c respectively.

$$\therefore \quad n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c \quad ...(1)$$

Thus, if fundamental units of both systems, dimensions of the quantity, and its numerical value n1 in one system, are known then we can easily calculate n2 in another system.

(c) Derivation of a relationship between various physical quantities is based on the principle of homogeneity of dimensions.

Following are the steps used:

- We must Know the physical quantities (say p, q, r) upon which a physical quantity say x depends.
- We must know the dimensions of p, q, r say a, b, c respectively.
- Then we write $x = p^a$
- Now, write the dimensions of each physical quantity on both sides of the equation
- and compare the powers of M, L, T to find a, b, c. Putting values of a, b, c in the equation
- we get the required relation.

Assertion Reason Answer:

1. (c) Assertion is correct, reason is incorrect

Explanation:

Dimensional constants are not dimensionless.

2. (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

Explanation:

As the distance of star increases, the parallax angle decreases, and great degree of accuracy is required for its measurement. Keeping in view the practical limitation in measuring the parallax angle, the maximum distance of a star we can measure is limited to 100 light years.

Case Study Answer:

1. i (d) Either (a) or (b)

Explanation:

A screw gauge and a spherometer can be used to measure length accurately as less as $10m^{-5}$

ii (c) Spring oscillator

Explanation:

Spring oscillator cannot be used to measure time intervals.

iii (a) 4.9 cm

Explanation:

Given, length, l = 5cm Now, checking the errors with each options one-by-one, we get

$$\Delta l_1 = 5 - 4.9 = 0.1 \text{ cm}$$
$$\Delta l_2 = 5 - 4.805 = 0.195 \text{ cm}$$
$$\Delta l_3 = 5.25 - 5 = 0.25 \text{ cm}$$
$$\Delta l_4 = 5.4 - 5 = 0.4 \text{ cm}$$

Error Δl_1 is least.

Hence, 4.9cm is most precise or accurate.

iv (a) $22.0cm^2$

Explanation:

Area of rectangle, A = Length´ Breadth

So, A lb = 10. × 5 = 22. $05cm^2$

Minimum possible measurement of

scale = 01. cm.

So, area measured by scale = 22 0. cm^2

v (c) 8.6 s

Explanation:

Magnification in time $= \frac{\text{Age of mankind}}{\text{Age of universe}}$

$$= \frac{10^6}{10^{10}} = 10^{-4}$$

Apparent age of mankind $= 10^{-4} \times 1$ day

$$= 10^{-4} \times 86400 \text{ s}$$

$$= 8.64 \text{ s} \approx 8.6 \text{ s}$$

2. i (a) 2

Explanation:

As, we know that the terminal or trailing zero(s) in a number without a decimal point are not significant. So, 4700m has two significant figures.

ii (b) ab × 10^b

Explanation:

Every number is expressed as ab × 10^b, where a is a number between 1 & 10 and b is any positive or negative exponent (or power) of 10.

iii (c) 4.8 g cm^{-3}

Explanation:

There are 3 significant figures in the measured mass whereas there are only 2 significant figures in the measured volume. Hence, the density should be expressed to only 2 significant figures.

$$\text{Density} = \frac{5.74}{1.2} = 4.8 \text{ g cm}^{-3}$$

iv (a) Change in unit does not change the significant figure.

Explanation:

There is no change in number of significant figures on changing the units. For it, the convention is that we write,

4700 m = 4700 × 10^3m

This convention ensures no change in number of significant numbers.

v (a) I and II

Explanation:

Following rules of significant figures are

I. All the non-zero digits are significant.

II. All the zeroes between two non-zero digits are significant, no matter where the decimal point is, if at all.

III. The terminal or trailing zero(s) in a number without a decimal point are not significant. Thus, 123m =12300cm =123000 mm has three significant figures, the trailing zero(s) being not significant.